

# THE MATHEMATICS TEACHER

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## RECENT ADVANCES IN THE TEACHING OF MATHEMATICS.

BY R. H. HENDERSON.

In attempting a discussion of our subject, we are confronted by three possible lines of attack: First, what advances, if any, in the subject matter that is presented in the ordinary courses in mathematics; second, what improvements are to be noted in the methods of presentation of mathematical subjects to the classes; and third, what advancement is worthy of note among teachers of mathematics as to their professional training and fitness to be recognized as leaders in their chosen profession. Any one of these lines of thought is capable of extended discussion which exceeds the limits of this paper. We shall, therefore, set forth under each some points which appeal to us as worthy of presentation on a subject of such vital interest to us all.

As to the subject matter of mathematics, we may say that there has been but little, if anything at all, put forth in recent years, that can be classed as entirely new. A writer on geometry asserts that there has been nothing new in that subject for the past two thousand years; that we today are following practically the same line of development as did Euclid. But be that as it may, that there has been advancement is evidenced by the fact that emphasis is placed upon certain topics that are more important and vital in their relation to the subject of mathematics as a whole, and to the sciences that depend upon mathematics for their mastery. Emphasis is now placed on those

things that are in the main essential, practical, and fundamental to the industries and to the sciences, to the exclusion of those subjects that are unessential and impractical. There is, therefore, no longer use for such topics as circulating decimals, alligation, cube root, continued fractions, and others that might be mentioned, because the pupil's time can be better spent on subjects more useful to him.

We mention first that emphasis is now being placed on the complete mastery of the equation as the great instrument for use in all succeeding mathematical work. If a pupil is to be well prepared for successful work in the extensive fields of engineering, he must be able to handle equations rapidly and efficiently. He must take his equations in their complex forms as given, simplify them, reduce them, transform them when necessary, in order to find the values of the unknowns. In fact the whole subject matter of the algebra centers around the handling of the equation. We add, subtract, multiply, divide, factor, simplify fractions, find powers and roots, and learn other important processes, not so much for their value as a knowledge of how to do them, but rather that we may be able to deal effectively with equations of all kinds, whether simple, quadratic, or of higher power. And we as teachers fall short of doing our full duty if we fail to place proper emphasis on a complete mastery of the equation.

The subject of determinants calls for brief notice because of its value to all who extend their reading and study of mathematics beyond the elementary fields, and because of their great value in the solution of equations containing four, five, six, or even more unknown quantities. We place emphasis upon a knowledge of the determinant notation because of its relation to the higher mathematics, and because of its power and utility as an instrument in research work.

Another advancement is the correlation of algebra and physics. This has arisen from the fact that science teachers have complained that their pupils could not handle physical formulæ because they did not know their algebra. In reply, the teachers of mathematics asserted that their work had been well done, and that the fault lay not with the pupils, but with the science teachers in the presentation of their subject. In truth,

each was partly in the right and both were in the wrong. The pupils had been taught to solve for  $x$ , or  $y$ , or  $z$ , as the case might be, and had thus fallen into a certain habit of thinking algebraically. When physical problems came to be considered, it presented something entirely new to their way of thinking and consequently confusion naturally arose. To overcome these difficulties, our textbooks now contain a few pages of physical formulæ, such as  $s = \frac{1}{2}gt^2$ ,  $C = E/(R - r)$ ,  $K = \frac{1}{2}(mv^2/g)$ , etc., and problems relating to them. By solving for the unknown value and then making the required substitutions, the pupil gets away from the routine way of doing everything in  $x$ ,  $y$ , or  $z$ . He is led to see that algebra is not only an end in itself, but also a means to successful work in physics, chemistry, and in the broader field of mechanics.

A third advancement in recent years is the introduction into elementary algebra of the subject of graphs, including the plotting of equations, the graphic solution of equations, and graphic analysis. Coming as it does from the analytic geometry, it was introduced into the elementary algebra as a preliminary step to that important subject. But its real value was soon appreciated as a separate subject even to those who might not pursue their mathematics further, because it enabled the pupils to have a much broader comprehension of the meanings attached to  $x$  and  $y$ . Abstract as much of our mathematical reasoning is, it can be made very concrete, and made to appeal strongly to the intellect, if we can visualize the conditions of a problem. For that which can be presented to the eye will always make a deeper and more lasting impression on the mind. We have all read a great deal of the events that have happened in Europe during the past eighteen months. But how eagerly do we look at a picture that portrays to us conditions as they really exist. By reading we get a general idea from which we build up our own individual mental images. But the picture gives a far more vivid impression. So in our teaching, mathematical relations may be discovered, or a truth driven home more forcibly, when by the use of the crayon or pencil a few lines are drawn, setting forth the conditions as they exist in some particular problem. Our pupils can solve rapidly such simple equations as  $x + y = 3$  and  $x - y = 5$ , and get the values of  $x$  and  $y$ , but

surely they will get a much broader conception of what we mean by simultaneous equations when they learn that the values of  $x$  and  $y$  are the co-ordinates of the point of intersection of the graphs of these two equations. In solving the equations of higher degree, the results can be more easily determined and better understood, if they know that roots of the equation are the points of intersection of the graph of the equation with the  $x$ -axis or the  $y$ -axis, as the case might be. The use of graphs is also useful and practical in many other ways. The statistician uses them to present his facts and figures to the eye in a more telling manner; the broker and the merchant use them to record the rise and fall of prices; the physician to record the progress of disease; the corporation to compare its work from year to year. So we see that graphs have won their place not only in a well-balanced course in mathematics, but in the industrial world as well.

Another subject in which rapid advancement has been made in recent years is trigonometry, and we note a decided improvement not only in the manner in which the subject is developed and presented but also in the devices by which trigonometric formulæ may be derived and remembered for future use. The greatest advancement has been in the matter of textbooks on this subject. A textbook of 25 to 30 years ago contains but little more than the simple facts of the subject presented in a dry uninteresting manner. Recent textbooks present the subject in a more interesting and attractive manner, and develop the subject in a more effective way. As to devices which add to the value of the subject as aids in using the formulæ more readily and in remembering them, we mention one or two. In dealing with the functions of an angle, it is often necessary to pass from one function to another and to do it quickly. As, for example, having given the  $\tan a/b$ , and we wish to know the sine, cosine, or any other function, for ready substitution. This can be obtained very quickly by actually drawing a right triangle, placing  $a$  as the side opposite to the given angle, and  $b$  as the side adjacent to the given angle, and then solving for the undetermined hypotenuse by the Pythagorean theorem. It is then possible to obtain any function of the angle at once and be sure you are right, without having to make use of the ordi-

nary transformations. This device is especially valuable in the handling of equations involving the inverse functions. We mention, also, a method by which it is fairly easy to remember the formulæ for the sum of the sines, for the difference of the sines, for the sum of the cosines, and for the difference of the cosines of two given angles. It is based on the well-known formulæ,  $\sin (x + y) = \sin x \cos y + \cos x \sin y$ , and  $\cos (x + y) = \cos x \cos y - \sin x \sin y$ , and in the order given. We recall that the coefficient in each case is 2, and that each formula involves one half the sum and one half the difference of the two angles. It remains for us to use the terms sin and cos in their proper places. The first term gives us sin and cos for the sum of the sines; the second, cos and sin for the difference of the sines; the third, cos and cos for the sum of the cosines; and the fourth, sin and sin for the difference of the cosines, with also the — sign in its proper place.

Under the subject of *method*, the greatest advancement in recent years is to be noted in the adoption of the so-called laboratory method, a term which is applied to any and all proposals that have for their underlying principle the one thought of adaptability and interest. The success or failure of any method is determined by whether or not it can be used to rouse the interest and hold the attention of the child. The laboratory method is good then, as judged by this standard, whether applied to the subject matter that we teach to our pupils, or the manner in which it is presented. By drawing upon subjects either in algebra or arithmetic with which our pupils are in a measure familiar, a lively interest is awakened, and they are ready to solve problems that arise out of their own experience, or are taken from topics about which they are accustomed to think. This means, then, that the problems should be real problems, drawn from fields of actual experience, in contrast to problems that are purely artificial or appeal only because they are humorous or ludicrous. We do not censure high-school pupils who do not become interested in many of the old "chestnuts" that are the heritage of the past, or problems such as this which are worthy of nothing more than a smile: "A woman walks 60 miles in 17 hours, walking 3 miles an hour uphill, and 4 miles an hour downhill. How many miles does she walk uphill and how many downhill?"

The sources of real problems are limited only by the pupil's individual interests. The boy on the farm will see in his arithmetic and algebra a means to a very important end if the work he is required to do is decidedly agricultural in its selection. The city boy will be all eyes and ears for his algebra if his teacher can make it plain to him that the large bridge over which he passes so often first existed in some one's mind, and that the same  $x$  and  $y$ , so often puzzling to him, had played their part in determining the stresses and strains, and the peculiar part each separate piece of iron and steel would have to endure in the completed structure. Our girls will not be continually asking the question, "What is the use of the study of algebra anyway?" if in our presentation of the subject we can cut loose from time-worn problems, and draw somewhat upon subjects that have a vital interest to girls in particular.

We are ready to take a more advanced position than this, based upon observation and study of conditions as we find them coming under our experience. There are certain facts to which we all can readily give assent. First, that our pupils do not all have the same mathematical ability; second, that all our pupils are not preparing for the same pursuits in life, and hence do not need the same preparation; third, that very many of our pupils under the best training possible to give them will be prepared to fill only mediocre positions at best, hence their training should be along lines that will better fit them for the greatest measure of success. Therefore, we are ready to assert that algebra as now required by commercial students is largely a waste of time from the standpoint of utility. We fully realize that algebra has its place for cultural value and mental training. And we would not lower in the least the amount of work required to take its place, but rather substitute in its stead a rigid course in commercial or industrial arithmetic, and our pupils will get the same amount of training and at the same time be better fitted for the commercial and industrial world. If we do not argue for such a change as this, how can we answer the challenge of all those pupils who leave our public schools and seek the business training in the private commercial school. Some one will say: Will not such a plan prepare our boys and girls for only one thing, and thus make them the victims of class education which will differentiate between them and those capable of greater

educational possibilities? But would it not be far better for our schools to give the best training possible to those whose capabilities are more limited than others, so as to fit them for a larger measure of success? The laboratory method, then, will not only aid us in determining largely how we shall teach, but also what we shall teach, so as to make mathematics a stepping stone to larger usefulness, rather than a stumbling block in the way of intellectual attainments.

But perhaps the best advancement of all, and the one which has been largely responsible for all the others we have named, is the improvement in the personnel of the teachers of mathematics themselves. And what we say on this point is applicable to the other members of the teaching profession as well. Teaching is no longer a step to other professions, because school boards have made it possible for those who engage in it to consecrate themselves wholly to this work by making the remuneration somewhat in keeping with the measure of service required. We have, therefore, met together as a body of teachers, not so much to hear papers read and discussed, as to exchange views each with the other, and find out what others are doing in the same lines of work as we ourselves are engaged. By doing so, we keep from getting into a rut and following the line of least resistance. The true teacher of mathematics will see growth and development in himself as well as in the pupils who sit in his class from day to day. It is a law of nature that life must spring from life, and surely we want that our pupils should drink from the living fountain rather than the stagnant pool. This means then, that the teacher of algebra, or geometry or trigonometry, should know more mathematics than just the subjects he is expected to teach. If he has studied the calculus, he knows that emphasis must be placed on factoring and the reduction of fractions in the algebra, and on remembering of trigonometric formulæ. If he would study function theory for a time, he would be more considerate of his pupils when they come to him puzzled over things which seem insignificant and which he feels ought to be mastered with little effort. And so, in conclusion, let me urge that the only true advancement is that which springs from consecrated teachers who are willing to prove all things and above all, hold fast that which is good.

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# TO PLOT $ax^2 + bx + c = 0$ .

By J. L. PATTERSON.

$x = -b/2a \pm \sqrt{(b^2 - 4ac)}/2a$ , which may be written  $x = -(b/2a) \pm k$ , from which it is evident that  $x = -b/2a$  is the equation of the axis of the curve, and if  $-(b/2a)$  be substituted for  $x$  in the equation  $ax^2 + bx + c = y$ , the value of  $y$  thus found will give the intersection of the curve with the axis of the curve, that is the lowest (or if  $ax^2$  be negative, the highest) point of the curve. *Thus the two most important characteristics of the curve, viz., the axis and the highest or lowest point are determined at the outset in a very simple manner.*

For example take the equation  $x^2 - 3x - 18 = 0$ . Use the equation of the axis  $x = -(b/2a) = \frac{3}{2} = 1\frac{1}{2}$ , which gives the axis of the curve, and if  $\frac{3}{2}$  be substituted for  $x$  in the equation  $x^2 - 3x - 18 = y$ , we find  $y = -20\frac{1}{4}$  which gives  $C$  the intersection of the curve with the axis of the curve, or the lowest point on the curve, and thus the two most important characteristics of the curve are known.

If  $x = 0$ ,  $y = -18$ , as usual, which gives the point  $D$  and the symmetric point  $D'$  is at once known. The solution of the equation gives  $x = 6$  or  $-3$  which gives the points  $E'$  and  $E$  as usual, and we have five points which would be sufficient for a rough plot. But we know the axis and the lowest point which are of vital importance.

If the curve does not cut the axis of  $x$  this method gives only three points but other points may be found in the usual manner, if necessary, which would seldom be the case.

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## ENTRANCE REQUIREMENTS AGAIN.

BY S. S. KELLER.

The writer has little hope of seeing and not the least desire to see this protean pedagogical question of entrance requirements completely answered to the purring satisfaction of everybody or of even a majority.

After all it is the unsolved problem that keeps us on the qui vive. And let us have no compromises—at least not yet.

In confidential truth, it is the more or less exigent presence, is it not, of these pedagogical offspring, obstinately refusing to be put to bed, that keeps many of our solemn conferences from becoming strong rivals of the church services for the sedative championship?

As a matter of fact the characteristics of any specific set of entrance requirements are very largely determined by the ideals of the institution employing them, and still more by the seriousness with which that institution takes its ideals. Since such ideals are various, have never been standardized, and are subject to fluctuations, the probability of a universal creed fixing entrance requirements is about as great as that of a universal brand of politics.

However while we doubtless will fail to settle anything, the holding of the subject up to the light and the turning of it about always improves its definition and gives us all a chance to accelerate our pedagogical circulation by mutually hooting each other's opinions.

For the purposes of this brief discussion, then, let us say that schools of higher learning so called, may be divided roughly into three classes: First, those having no ideals that will not speedily evaporate upon exposure, or only those of such versatility as to fit almost any condition. Second, those that have, card indexed, a complete set of excellent ideals of highly respectable lineage. They never cause any excitement, but are often admired by eminent visitors. Third, those that have a virulent attack of ideals (that often keeps the temperature rather high) and be-

lieve in these ideals fiercely, and it is to be feared, at times combatively.

With the first class this discussion is very little concerned. The chief ambition of such institutions is to acquire merit in the educational census reports, their annual estimates of themselves being often remarkable contributions to that *sui generis* but always vivacious form of fiction known as "catalogues." No form of entrance requirement whose meshes were not large enough to admit anything numerically available could be popular with them. Happily they are few.

The second class comprises those schools that have a collection of thoroughly respectable ideals of hallowed memory, but nothing really vital roots in them; there is no cult, so to speak, based on them. If students come to them, worthy students, they will of course conscientiously lead them along the conventional paths to wisdom, but they are not convinced that their guidance is distinctly superior to that of any other institution whatsoever, but they are sure that he will not arrive out of breath. Any form of entrance requirement that is pedagogically genteel will suffice. Examinations are usually required but they are apt to be chastely conservative with something of the flavor of a sacrament.

It is much to be feared that the explorations of these institutions into the mental preserves of aspiring youth throw but a faint and fugitive light upon the problem of estimating their potentialities for educational progress.

The third class of institution is profoundly interested in the solution or in any worthy approximation to a solution of this problem.

As intimated it has dreams and they are very vivid and disturbers of the peace. It believes in the reality of these visions almost with ferocity. It is convinced that it has a genuine pedagogical mission, is on the trail for converts and doesn't care who knows it.

Also it may readily become a nuisance.

It is from the point of view of these restless parties that the writer would like to say what he may have to offer on what seems to him a desirable system of entrance diagnosis.

It is of course unnecessary to remark that an institution or an

individual that carries a high pressure of enthusiasm into an intellectual territory already well occupied is bound to lose cuticle and to acquire chastening experiences.

A casualty list of theories are of no interest here. Those that survived are the following:

There are three lines of inquiry in the attempt to diagnose the intellectual fitness of an applicant for admission to our educational midst: first, we want to know what sort of intellectual provender has been supplied and in what quantities; second, how much of this mental food has been digested and assimilated; third, what capacity has the candidate for further absorption and assimilation.

A fairly satisfactory answer to the first can be obtained from the preparatory school certificate, at least we find it so. To us this is the least important part of the triple inquisition, although it has a certain illuminating value; is in fact indispensable. Perhaps I should say that it is least important because it is so readily and definitely determinable.

The second item of information may we think be acquired to a fair degree of approximation at least, by a carefully prepared examination; one that not only tests the student's storage capacity but, as far as may be, his intellectual metabolism, if I may be permitted such verbal atrocity.

The most ingeniously devised and adroitly expressed examination will of course fall much short of realizing this ideal, but it helps our third inquiry.

I should like to say in passing that I fear secondary education lays an undue stress upon the importance of an agile and retentive memory. At any rate the student too often is obsessed with the idea that his admission to the questionable delights of a "higher education" depends very seriously upon his ability to set aside a large portion of his mental establishment as a mere temporary depot for facts, which he must keep in storage until he has passed his entrance exams. I am not at all sure that certain forms of entrance examination do not justify this attitude, but I deplore it none the less. This, however, is quite another tale.

Finally we should like to know what capacity the applicant may have for further absorption and assimilation.

Unfortunately the entrance examination has very little to say upon this most important question, and no instrument of precision has yet appeared that can rescue its solution from the well-founded suspicion that clings to purely subjective judgments. Modern psychology has heroically assumed the task of exploring the intellectual organisms of wisdom seekers, but it still remains to be seen what value will accrue from their efforts. It is to be hoped that the adventuring of psychology into this dimly-lighted territory may not result in its getting lost in a jungle of metaphysical subtleties. For it seems to the writer that this last inquiry into the preparation of the prospective student is much the most important.

For want then of a better method of procedure we have adopted what is known to us as the "personal interview," in the effort to locate intellectual leaks and limitation. It consists in a quiet talk as intimate and unconstrained as possible, wherein the candidate's clarity of view and expression along several lines is tested without his being aware of it. The success of such a process obviously depends entirely upon the tact and the acumen of the interviewer. If his method of approach is crude he will unmask his battery and put the student on his guard too much to secure accurate data from him. Besides he must be a keen and accurate observer of human nature especially in its juvenescent form. Sounds like a rather large contract doesn't it? But a first-class teacher ought to possess these traits ought he not? And besides no system of such sort could be expected to reach high efficiency without mishaps, or without passing through a period wherein its supporters did feel like taking to cover.

Each applicant is given a rating as a result of this beneficent "third degree," and this rating counts full share in the final decision as to the admission of such applicant. Of course after a few years of such practice results help much in checking up the fidelity of the interviewer's estimates, and it also improves his subsequent judgments.

A series of tests are also to be made by the department of psychology, which will be compared with the conclusions of the interviewers. It will at least be an interesting comparison and subsequent developments will be apt to show somebody up in

the line of vaticination. Prophesying is parlous and highly unprofitable work, but we all will take a fling at it now and then. Well this is not a prophecy, it is a sort of premonition, namely, if psychology makes good in getting mental X-ray pictures of applicants for admission to college or technical school, I believe that the examination test will be left wholly in its hands.

Neither do I believe there would be many mourners at the obsequies of the old-style examination, that bugaboo of student and unpopular assistant of faculty.

To an institution that believes devotedly in its mission, the question of entrance requirements at best is a perplexing and exigent problem. It is most anxious to secure abundant material in the shape of students upon whom to work its educational miracles, and yet it does not want to clog up its machinery with inept and hopeless specimens that will imperil its standards. It must find a dignified position between repellant exclusiveness and vulgar laxity. Will some brother kindly rise and tell us what that is?

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## THE USE OF THE RADICAL SYMBOL.

BY G. A. MILLER.

Probably the most significant recent step towards the uniformisation of the notation of elementary and secondary mathematics is represented by the list of symbols, numbered from 1 to 139, proposed in Heft 17 of the *Schriften des Deutschen Ausschusses für den mathematischen und naturwissenschaftlichen Unterricht*, 1913. In many cases remarks and explanations are added to the proposed symbols. Number 20 of these symbols relates to an  $n$ th root, and, as this may be of special interest, we shall reproduce it here, translated into English.

20  $\sqrt[n]{a}$   $n$ th root of  $a$ .

*Remark 1.*—The use of the parenthesis in place of the stroke is to be avoided when possible.

*Remark 2.*—The notation  $\sqrt{a}$  without the stroke is allowed when the root symbol is followed by a number represented by Hindu-Arabic numerals or by a single letter.

*Remark 3.*—The root index should be written within the root symbol. Thus  $\sqrt[n]{a}$ , not  $n\sqrt{a}$ .

*Remark 4.*—In case of the square root the index is commonly omitted.

*Remark 5.*—The  $\sqrt[n]{a}$  represents the positive number whose  $n$ th power is  $a$  when  $a$  is positive. When  $a$  is negative and  $n$  is odd then  $\sqrt[n]{a}$  represents the real number whose  $n$ th power is  $a$ .

*Explanation.*—The single-valuedness of the root symbol noted in Remark 5 seems to offer the only possibility of avoiding inexactness in teaching students in the schools. It is worth noting that the equations

$$a^3 = 1 \text{ and } a = \sqrt[3]{1}$$

as well as the equations

$$a^2 = 1 \text{ and } a = \sqrt{1}$$

are not regarded as identical according to the said remark. The former equation in each of these two sets of equations is for the time being to be regarded as multiply-valued, while the latter is single-valued. The confusion is especially serious in the introduction of logarithms, if we let

$$\sqrt[n]{a} = a^{1/n}$$

and then transform

$$a^{1/n} = b \text{ into } \frac{1}{n} = \log_a b$$

without having first stated the single-valuedness of  $\sqrt[n]{a}$ .

While the said restrictions on the meaning of the radical sign tend to definiteness they evidently do not touch some of the most serious difficulties. Even in our elementary algebra courses we are concerned with square root of negative numbers. For instance, the roots of the quadratic equation

$$ax^2 + bx + c = 0$$

are commonly represented in the form

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The question arises what does

$$\sqrt{b^2 - 4ac}$$

mean when  $b^2 - 4ac$  is negative or complex. We should clearly not speak of a positive and a negative square root in this case since the terms positive and negative relate only to real numbers, being short expressions for argument of  $0^\circ$  and  $180^\circ$  respectively. On the other hand, it is customary to consider  $\sqrt{-1} = i$  as situated on the positive part of the  $y$ -axis, and it would appear desirable to add to Remark 5, noted above, that  $\sqrt[n]{a}$ , when  $a$  is negative and  $n$  is even, represents the number whose argument is  $\pi/n$  and whose  $n$ th power is  $a$ .

If this were done the symbol  $\sqrt[n]{a}$  would have a unique meaning whenever  $a$  is real, and  $n$  is a positive integer, and the students of elementary algebra could be taught a single meaning of this symbol which would suffice for the solution of the quadratic equation with real coefficients. It is true that this definition of the symbol  $\sqrt[n]{a}$  has the disadvantage of lacking generality, since when  $a$  is negative we define it in two different ways as  $n$  is odd or even, but if the difference of these definitions were properly emphasized it would tend to clearness in the use of the radical sign in our elementary work.

The question whether it would be desirable to define the symbol  $\sqrt[n]{a}$  uniquely also when  $a$  is any complex number is not pressing so much for an answer. In fact, desirable definiteness would be secured by stating that this symbol has a unique meaning only when  $n$  is a positive integer and  $a$  is real. When  $a$  is complex it would clearly be possible to define  $\sqrt[n]{a}$  uniquely by saying that it represents the number whose argument is  $1/n$ th the argument of  $a$  and whose  $n$ th power is  $a$ , and thus secure uniformity with the case when  $a$  is negative and  $n$  even. With such restrictions on the radical sign it would have distinct advantages over the corresponding fractional exponent notation and there would appear substantial reasons for continuing the use of the radical sign in elementary algebra.

To avoid the possibility of an incorrect inference it is desirable to add that in the "explanation" quoted above we represented  $\log b$  to the base  $a$  in the usual form  $\log_a b$ . In said list this is written in the form

$$^a\log b$$

in accord with a notation suggested by A. L. Crelle in his "Sammlung mathematische Aufsätze," Vol. I, 1821, page 207, and it is suggested that the base should always be noted unless it is 10 or  $e$ . In many German publications the base is written above the abbreviation of the word logarithms, thus

$$\log^a b.$$

The form  $\log_a b$  is commonly used not only in English, French, and Italian, but also to some extent in German, and hence it would appear that its use should become universal. It seems unfortunate that this form does not appear in said list of suggested symbols.

It should perhaps also be explicitly noted that the logarithm of every number has really an infinite number of values as was proved already by L. Euler. In using logarithms of positive numbers it is customary in elementary mathematics to confine our attention to the single real value. In this way  $\log a$  becomes a single-valued function of  $a$  whenever  $a$  is a positive number. The given suggestions as regards the single value of the radical are therefore in accord with other restrictions in our early

mathematical work, and such suggestions have been advocated many times. In fact, in his noted "Analyse algébrique," 1821, A. L. Cauchy distinguished for the first time the principal values of elementary functions by a convenient notation, using the symbols

$$l(z), \quad a^z, \quad \sqrt[n]{a}, \quad \text{arc sin } z$$

for the principal values, and the symbols

$$l((z)), \quad ((a))^z, \quad \sqrt[n]{a}, \quad \text{arc sin } ((z))$$

for the corresponding general values of these expressions.\* This notation has, however, not been widely adopted and we still find too frequently the same notation used both for the principal value and for the general values.

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\* "Encyclopédie des Sciences Mathématiques," tome II, Vol. 2, p. 50.

## MATHEMATICS APPLIED TO DOMESTIC ARTS.

BY KATHARINE F. BALL.

A diligent search in mathematics textbooks that offer even the slightest promise of the application of mathematics to domestic arts, reveals how little attempt has been made to discover the part that mathematics plays in what has come to be considered peculiarly "woman's sphere." This is not because the domestic arts do not offer an opportunity for the application of mathematics, but because of general social and educational conditions that have affected the kind of training given to girls.

During the past few years scattering problems on foods and dress-making have crept into some of the so-called practical arithmetics and algebras, but there has been little attempt to gather such problems together into a systematic course for girls.

That is what we have been trying to do in our course in household mathematics in the Plainfield high school. The aim of this course is two-fold: first, to emphasize the economic aspect of household problems, and, second, to make the girls skillful in solving the problems of the home. These two aims have determined the character of the course, the order of topics, and the mathematical content. We have tried to select only problems similar to those that actually occur in the home.

These problems have been grouped, not according to their mathematical content, but according to subject matter, the order of topics depending to some extent upon the order of topics in the home arts course, to some extent upon the difficulty of the problems, and to some extent upon the maturity of the girls. Under our present arrangement the course is allowed 7 credits, of which 5 are given in the sophomore year and 2 in the senior year.

In the sophomore year the course begins with a study of the budget system as applied to household management. This lays a foundation for the economic aspect of all the later work. The girls learn what a budget represents, how to plan a family budget, and how to apply the theory to their own personal ex-

penditures. They are taught to keep a petty cash account, to distribute the items of expenditure under the budget headings, and to see that they keep within the budget estimates.

The budget divisions in a sense form an outline for the rest of the course. If we accept Bruère's choice of headings, the subjects taken up may be grouped as follows: food, shelter, clothing, operation, and advancement. At present we study food, clothing, and operation during the sophomore year, and shelter and advancement during the senior year. Whether or not this is a desirable arrangement is irrelevant to our present discussion.

The senior course offers little in the way of new problems, though it may be novel to consider them women's problems. The girls make an intensive study of methods of keeping household accounts, and they study investments, methods of purchasing homes, building and loan associations, mortgages, life insurance. They compare the expense of owning a home, such as loss of interest on investment, taxes, depreciation, etc., with rent of similar property. The maturity of the seniors makes it possible to study all these problems from a more or less personal point of view.

In the sophomore course the problems peculiar to domestic arts find their place. After the preliminary study of the budget system already mentioned, the first problems to be considered are those in operation, because they present fewest difficulties. These problems are concerned chiefly with matters of fuel for heat and light. The girls read the various kinds of meters in the school. They find out the difference in the number of feet of gas used per hour in a Welsbach and in an open burner; the reason why it is cheaper to use a gas iron rather than an electric iron; the number of hours of use needed to make a gas iron pay for itself; they discover why one cannot afford to use the ordinary carbon electric light bulb instead of a tungsten or Mazda. All these problems, though extremely simple as far as mathematics are concerned, have a legitimate place in the course because of their importance to the housekeeper.

Closely connected with operation is the matter of house furnishing and the buying of supplies. All the variety of problems in this group will readily occur to any housekeeper's mind:

estimating the amount of material needed for floor coverings, wall paper, table and bed linen, curtains, etc. We give the girls a little practice in drawing to scale, and teach them how to draw floor plans and interpret architect's drawings. The economic aspect of the problems is emphasized wherever possible, both in studying the relation that the value of furnishings should bear to the value of the house, and also in pointing out the significance of even a small saving through canny methods of purchase. *E. g.*, kitchen soap purchased by the cake at 5 cents or in quantities at 4 cents means a possible saving of 25 per cent. If the same per cent. of saving could be realized on all purchases what would it amount to in an outlay of \$400?

The problems in house furnishings involve only denominate numbers, percentage, and mensuration of rectilinear figures and the circle. The circle is needed for such a problem as this:

How much lace is needed to edge a circular lunch cloth one yard in diameter?

In the next division of the budget, clothing, are included all problems concerned with estimating the amount of material needed for garments, the allowance for hems, tucks, straight and bias ruffles, and the cost of the same. These problems involve only fractions, linear measure, and square root, but I can assure you that they are sufficiently puzzling for an ordinary high-school sophomore.

To illustrate: A tuck shortens the goods by double the width of the tuck. Problem: How many  $\frac{1}{4}$ -inch tucks are needed to shorten a skirt 3 inches? The skirt is to be 38 inches long, finished. How long must each breadth be cut to allow for a 3-inch hem and three  $\frac{1}{4}$ -inch tucks?

Again: A ruffle should measure  $1\frac{1}{2}$  times the length of the goods to which it is attached. Problem: A child's petticoat measures  $1\frac{1}{2}$  yards around the bottom, and is to be 16 inches long finished, with a 1-inch hem. How much material 30 inches wide is required? If it is trimmed with a ruffle 3 inches wide, finished, that has a  $\frac{1}{2}$ -inch hem and three  $\frac{1}{8}$  inch tucks, how much will be needed for the ruffles? Will any extra material be needed for the band? At 35 cents a yard, how much will the petticoat cost?

Bias ruffles add a new difficulty. For the benefit of the men

may I explain that a "true bias" is in reality the diagonal of a square whose side is the width of the goods. A bias strip is made by cutting parallel to the diagonal. The width of a bias strip is the perpendicular distance between the lines of cutting.

To estimate the length of a bias strip the dressmaker's rule is: multiply the width of the goods by  $1\frac{1}{2}$  (approximately  $\sqrt{2}$ ). In order to use this rule intelligently the girls learn how to find the square root of numbers, and how to make approximations.

Problem: Find the amount of material 20 inches wide, required for a 4-inch bias ruffle for a skirt 2 yards around the bottom, no strip to be less than 26 inches long. How much material will be wasted if the end pieces cannot be used? How much material would be needed for a straight ruffle?

If the material costs \$1.15 a yard, how much less will the straight ruffle cost?

When the subject of food is taken up, the problems become more complex, although the arithmetic involved is simple enough. The terminology has already been mastered in other courses; the girls know about the composition of food and its fuel value, they know what a great Calorie means as applied to foods, they know what is meant by a "balanced ration." But before this knowledge can be of real service they need to have a great deal of practice in applying the principles of nutrition to the housewife's problem of planning correct dietaries readily and economically.

To apply the principles of nutrition necessitates the use of tables that give the composition and fuel value of foodstuffs. Of the tables that are available, those compiled by Atwater and published as a government bulletin are the most important as well as the cheapest.

The Atwater tables give both the composition of foodstuffs in per cent., and also the total number of Calories per pound.

To familiarize the girls with the use of these tables they are given problems like the following, and they are required to tabulate their results in a convenient form for future use.

I. Find the total number of Calories and the number of Calories of protein, fat, and carbohydrates in 1 cup of rice.

1 cup of rice equals 8 oz.

Rice contains 8 per cent. protein

1 oz. of protein yields 113 calories

The problem thus reduces to simple multiplication of the factors 8 oz., 8 per cent., and 113 Calories. To find the number of Calories of fat the method is the same, except that fat yields 255 Calories to the ounce.

Problems in the comparison of foods are also given.

*E. g.*, how many ounces of sirloin steak will yield as many Calories of protein as one egg?

In such problems, the algebraic equation simplifies the solution, letting  $x$  represent the number of ounces of steak needed.

It is evident that the use of these tables involves more computation than is practicable for the average housekeeper. The information given is not in a form that makes it readily available for her use. What the housekeeper needs to know is not the number of Calories per pound, but per cup or ounce, not the chemical composition of foods in percentages, but the number of Calories of protein, fat, and carbohydrates per cup or ounce.

There are two other tables, either of which is more useful to the housekeeper than Atwater's; the table compiled by Professor Irving Fisher and reprinted as a bulletin by the American School of Home Economics, and the table compiled by Carlotta Greer and published in her "Textbook of Cooking." Both of these tables give the necessary information in a form that makes it readily available for the housewife's needs. Other admirable tables are those compiled by Locke, by Kinne and Cooley, and by Rose. But they are not so well adapted to use in the high school.\* While tables in which the gram is used as the unit involve simpler computations, they are not practicable because the ounce and the cup are the housewife's measures. In a short high-school course, it is useless to suppose that the girls can be taught to think in grams. Both Fisher and Carlotta Greer use the ounce as the unit, and both base their calculations upon the hundred calorie portion. Instead of the per cent. of composition, as in Atwater, or the weight of each of the digestible nutrients, both the Fisher and the Greer tables state the number of Calories yielded by protein while Fisher adds also the number of Calories of fats and of carbohydrates. From either Fisher's or Greer's tables it is comparatively simple to compute the

\* Since this article was written a new food table admirably adapted to use in the high school has been published by Professor Mary S. Rose in her "Feeding the Family."

number of Calories of any given food stuff, and of a few standard recipes such as rice pudding, white sauce, etc. When the recipe is not found in the tables, its fuel value has to be computed from its respective ingredients, and then it is sometimes necessary to know how to use the Atwater tables.

Although the actual mathematical principles involved in these computations are simple enough, it requires a great deal of practice to enable the girls to use the tables with any degree of skill.

They have to learn how to obtain approximate results, for approximations are really of more use to the housekeeper than the scientifically accurate results of the laboratory. In every way we try to simplify the methods of computation and to eliminate all but the essentials. Two main requirements only are considered in regard to dietaries: First, a sufficient total number of Calories, and second, the relatively correct number of protein Calories, thus assuming that if the per cent. of protein in the dietary is correct, the per cent. of fats and of carbohydrates can safely be left to adjust themselves. If the dietary needs correction because it is not balanced, the corrections are made entirely by trial.

The girls are expected to learn, both by practice and by actual memorization, the fuel value of certain common foods, *e. g.*, an egg, a slice of bread, a potato, a pat of butter, etc.; to know which foods can be used to increase or to decrease the per cent. of protein in a dietary; to know a certain list of combinations of foods that are practically balanced, *e. g.*, vegetable salads, bread and butter, etc. They are encouraged to make rough estimates of the fuel values of foods, recipes and dietaries, verifying their judgments by reference to the tables.

When the girls are sufficiently familiar with these fundamental principles and methods of dietetics, they have to consider also the economic aspect of the subject. Foods are then classified according to their cost per 100 Calories at the current local prices. The girls plan dietaries at a given cost per day, and learn how to lower the cost of living by choosing foods from the list of those that cost "less than 1 cent per 100 Calories." They find out which is the cheaper source of fuel at the current prices, eggs or steak, and they discover why tomatoes are a luxury as far as fuel value is concerned.

In all of this work we barely touch upon many of the important phases of dietetics: the ash constituent, the digestibility of food, the varying capacity of the different proteins for rebuilding tissue and sustaining life. All these, and many other complexities, have to be left for later and more intensive study in college. The aim of this part of the course has been achieved if the girls can use the tables readily, if they realize that it is not only desirable, but practicable for a housewife to plan meals scientifically, and if they are able to put their knowledge to use in planning balanced dietaries that have sufficient nourishment and variety and yet are within a specified cost.

This work in foods completes the sophomore course. An attempt has been made, by taking up the five divisions of the budget in the sophomore and the senior years, to include all types of household problems that involve mathematics in their solution. This course is still in the experimental stage, but the value of such training for the girls in the home arts course has been clearly demonstrated.

PLAINFIELD HIGH SCHOOL,  
PLAINFIELD, N. J.

# CIVIL SERVICE QUESTIONS IN MATHEMATICS.

BY LEONHARD FELIX FULD,

ASSISTANT CHIEF EXAMINER, MUNICIPAL CIVIL SERVICE COMMISSION,  
NEW YORK.

## PROMOTION TO ASSISTANT COURT CLERK.

[CITY MAGISTRATES' COURT.]

ARITHMETIC.

(Weight 15.)

1. Add: 6,475,869  
8,697,081  
7,586,453  
2,948,675  
5,465,768  
4,321,043  
9,293,949  
2,345,678  
9,999,999
2. The population of a city Jan. 1, 1915, was 123,450; during the year there were 2,469 births, 1,976 deaths, 1,258 people moved in and 701 moved away. What was the population Jan. 1, 1916?
3. Paid \$7,975 for land at \$55 an acre. Sold a part of it for \$3,625 at \$62.50 an acre, and the remainder at \$50 an acre. What was the gain or loss?
4. If iron rails weigh 384 pounds a piece, how many rails would be required for a track whose total weight is 1,105,920 pounds?
5. If the water supply for a city averages 3,456,789 gallons daily, how many gallons would supply it for 359 days?

## FIRST GRADE CLERK.

ARITHMETIC.

*(Weight 3.)*

1. A certain bank received and paid out the following amounts of money in one week:

	Received.	Paid Out.
Monday .....	\$150,267.50	\$ 99,812.56
Tuesday .....	85,072.81	122,917.88
Wednesday .....	199,768.57	75,856.24
Thursday .....	88,507.99	180,699.01
Friday .....	55,564.87	52,785.26
Saturday .....	101,868.17	88,802.17

Find the difference between the total receipts and payments.

2. A fence one mile long was under repair. On the first day,  $113\frac{1}{4}$  yards were completed; on the second day,  $86\frac{3}{4}$  yards; on the third day,  $99\frac{7}{12}$  yards; on the fourth day,  $83\frac{1}{2}$  yards; on the fifth day,  $170\frac{7}{12}$  yards; and on the sixth day,  $99\frac{7}{8}$  yards. How many feet of fence were unfinished at the end of the six days.
3. There are eight tenement houses on a certain block on Third Ave. The street floors are rented at \$50 each per month for business purposes. The four floors above are divided into four apartments on each floor, the rent of each apartment being, on the average, \$25 per month. If all the floors were tenanted and the rent was paid regularly, how much would the owner of the eight houses receive in rent each month?
4. A man bought a house for \$15,500 and spent \$900 on repairs. He then sold the house for \$17,220. Find the gain per cent. in this transaction.
5. A merchant borrowed from a bank a certain sum of money at 6 per cent. per annum. In 1 year, 8 months, 18 days he paid the bank in full, \$1,764.80. What was the amount of the original loan?

## JUNIOR DRAUGHTSMAN, [GRADE B].

*Duties:* Junior Draughtsman will be required to make sketches, tracings or drawings of an elementary character. They will assist in making maps, charts or diagrams and will perform computations incident to the work of draughting.

## MATHEMATICS.

(*Weight 2.*)

1. Add together the following:  $3' 9$  and  $\frac{5}{8}''$ ,  $2.75'$ ,  $18$  and  $\frac{3}{4}''$ ,  $6.5'$ ,  $4$  and  $\frac{3}{4}'$ ,  $106$  and  $\frac{15}{16}''$ ,  $75'$ ,  $9.5'$ ,  $49$  and  $\frac{7}{16}''$ ,  $2' 8$  and  $\frac{1}{4}''$ .
2. How many square yards of pavement are in a portion of a street  $36$  ft. wide with a length on one curb of  $128$  ft. and on the opposite curb  $139$  ft.? How many cubic yards of Macadam if  $9''$  thick?
3. A street starting at elevation  $36$  rises for a mile at  $1\frac{1}{4}$  per cent. grade and then for  $300$  feet falls at a  $2$  per cent. grade. What is the elevation at the end?
4. A standpipe of concrete is  $10$  ft. inside diameter,  $2$  ft. thick,  $15$  ft. high and has a bottom  $2' 6''$  thick. How many cubic yards of concrete are required to build it?
5. How many square feet of surface are there on a peaked roof  $60$  ft. long,  $30$  ft. wide, with a rise of  $9$  ft. and rafters projecting  $2$  ft.?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,

February 16, 1917.

DEAR PROFESSOR METZLER:

Professor J. W. Young—as chairman of the National Committee on Mathematical Requirements—has requested Professor D. E. Smith and myself to cooperate in the preparation of a report on the criticisms of mathematics, making a critical examination of the grounds of the more prominent and more responsible attacks on mathematics, with a view to determining the criticisms which are clearly not valid, those which are clearly justifiable, and those concerning the validity of which there is

reasonable doubt, with a view in the latter case to resolving the doubt if possible.

We desire, with your permission, to bring the matter to the attention of readers of the *Mathematics Teacher*, with the hope that they may assist us by bringing to our attention all possible material of value, particularly such as might otherwise escape attention. Communications on the subject may be addressed either to D. E. Smith, Teachers College, Columbia University, or to H. W. Tyler, Massachusetts Institute of Technology, Cambridge.

Very truly yours,

H. W. TYLER.

PHILADELPHIA, January 4, 1917.

TO THE MEMBERS OF THE ASSOCIATION:

Our territory is so large that the majority of our members find it inconvenient, if not impossible, to attend the spring and fall meetings of the Association. It is the object of the Council, however, to form Sections whenever a group of members are conveniently located near some town or city in which a Section of the Association might be expected to thrive. The Sections already organized in New York, Pittsburgh, Rochester, Syracuse and Baltimore usually meet two or three times a year. The meetings attract not only the local members of the Association, but also prove interesting to a larger group of teachers who have not yet identified themselves with us. The Section in Baltimore was organized at the last meeting of the Association, December 2, 1916. It starts with a nucleus of about fifty members and promises to do an important work in Baltimore, Washington, and the South.

We find that some of our members are not receiving notices of sectional meetings which they might attend at least occasionally. Such members are asked to send their names to the sectional officers or to the Secretary of the Association. All members of the Association may become members of the Sections upon application. For your convenience, the officers of the Sections are listed in this number of the *MATHEMATICS TEACHER*.

The officers of the Association are anxious to receive suggestions from any of the members in regard to the work of the

Association. Those wishing to present papers at the general meetings or at the sectional meetings, should notify the respective secretaries. In many professional associations there is a large supply of papers awaiting presentation, and we wish to impress our membership that the council will heartily welcome papers from any of our members who may have ideas meriting a place on our program.

Our work must be largely missionary. It is our duty to improve the teaching of mathematics from the kindergarten to the university. Every member of the Association should endeavor to interest in our work those who are not members and who may be benefited by joining the Association. We are more than five hundred strong, but in a territory as large as ours, and including school and college teachers, we should be five thousand strong.

The MATHEMATICS TEACHER reaches many who are not members of the Association. Its subscription list includes libraries from the Atlantic to the Pacific, as well as names of many leading teachers beyond our geographic boundaries. Perhaps some of your friends would become subscribers to the MATHEMATICS TEACHER even though they might feel unable at this time to join the Association. Our journal is undoubtedly the leading magazine in the high-school field devoted to the teaching of mathematics. There are other excellent journals which combine this with other allied purposes, but our journal is devoted chiefly to the pedagogical side of our work. We should have more subscribers, not primarily for their financial help, but because the MATHEMATICS TEACHER will exert a helpful influence upon all teachers of mathematics, and will prove to be a great stimulus and inspiration in their professional work.

Will you help us to make the coming year one of growth and greater activity for the better teaching of mathematics?

Very sincerely yours,

J. T. RORER.

## NEW BOOKS.

**Plane Geometry.** By EDITH LONG and W. C. BRENKE. New York: The Century Co. Pp. vii + 276. \$1.00.

This is the second book of the series "Correlated Mathematics for Secondary Schools," of which the first "Algebra—First Course" was reviewed in the issue of December, 1913.

The second book is concerned almost entirely with geometry, the algebra being confined largely to numerical and literal originals, and ratio and proportion. Trigonometry, however, receives considerable attention, and is carried into a few theorems on lines that usually are taken up in analytical geometry.

The best feature of the book, and it is a very important one, is its analyses of theorems. In its handling of the logical side of the subject it is far superior to the usual text.

**Calculus.** By H. W. MARCH and HENRY C. WOLFF. New York: McGraw-Hill Book Co. Pp. 360. \$2.00.

The authors of this Calculus have aimed to present the subject as a means of studying scientific problems rather than a collection of proofs and formulæ. The separation into differential and integral calculus is not maintained but the two are interwoven and carried together. Taylor's theorem and dependent topics are left until near the end of the volume. There is a chapter on solid analytical geometry and one on differential equations.

**The Algebraic Theory of Modular Systems.** By F. S. MACAULAY. Cambridge: The University Press. Pp. 112. \$1.10.

This is No. 19 of *Cambridge Facts in Mathematics and Mathematical Physics*. The chapter headings indicate the nature and scope of the work: The Resultant; The Resolvent; General Properties of Modules; The Inverse System.

**Automobile Repairing Made Easy.** By VICTOR W. PAGE. New York: Norman W. Henley Publishing Co. Pp. 1056. \$3.00 net.

Outlines every process incidental to motor car restoration. Gives plans for workshop construction, suggestions for equipment, power needed, machinery and tools necessary to carry on business successfully. Tells how to overhaul and repair all parts of all automobiles. The information given is founded on practical experience, everything is explained so simply that motorists and students can acquire a full working knowledge of automobile repairing. Other works dealing with repairing cover only certain parts of the car—this work starts with the engine, then considers carburetion, ignition, cooling and lubrication systems. The clutch, change speed gearing and transmission system are considered

in detail. Contains instructions for repairing all types of axles, steering gears and other chassis parts. Many tables, short cuts in figuring and rules of practice are given for the mechanic. Explains fully valve and magneto timing, "tuning" engines, systematic location of trouble, repair of ball and roller bearing, shop kinks, first aid to injured and a multitude of subjects of interest to all in the garage and repair business. All illustrations are especially made for this book, and are actual photographs or reproductions of engineering drawings.

This book also contains special instructions on electric starting, lighting and ignition systems, tire repairing and rebuilding, autogenous welding, brazing and soldering, heat treatment of steel, latest timing practice, eight and twelve cylinder motors, etc. You will never "get stuck" on a job if you own this book.

**Laboratory Manual for General Science.** By LEWIS ELHUFF. Boston, D. C. Heath and Company. Pp. vi + 90.

This manual is planned to accompany the author's "General Science, First Course." It contains 112 exercises, some to be demonstrated by the teacher, others by the teacher and pupils together, and still others by the pupils individually. The material seems to be well-chosen, both as to interest and usefulness.

**Synthetic Projective Geometry.** By DERRICK NORMAN LEHMER. Boston, Ginn and Company. Pp. xiii + 123. Price 96 cents.

The author has written a very interesting introduction to this subject. It avoids algebraic methods, and does not presuppose any knowledge of analytic geometry. The topics covered include correspondences, point rows and pencils of the first and second order, Pascal's and Brianchon's theorems, duality, poles and polars, involution, metrical developments, and the history of the subject.

**Practical Drawing.** By HARRY WILLIAM TEMPLE. Boston, D. C. Heath and Company. Pp. 141.

The purpose of this book is to teach eighth grade pupils to make practical working drawings, and to read blue prints. It is planned as an integral part of the course in shop work, and so has the advantage of making its usefulness clear to the pupil as the course progresses.

The applications are varied and within the capability of boys of this age, the plates are clear, and the whole book is well planned.

## NOTES AND NEWS

THE Annual Meeting of the Association of Teachers of Mathematics in New England was held in Boston, Saturday, December 9, 1916, at the Boston University College of Business Administration.

The following officers were elected:

*President*, Mr. Harry B. Marsh, Technical H. S., Springfield.

*Vice-President*, Professor Robert E. Bruce, Boston University.

*Secretary*, Mr. H. D. Gaylord, Browne and Nichols School, Cambridge. 104 Hemenway St., Boston.

*Treasurer*, Harold B. Garland, High School of Commerce, Boston.

*Members of Council*, Professor Helen A. Merrill, Wellesley College; Mr. Frederick E. Newton, Andover Academy.

The Secretary reported on a conference with the New England Association of Colleges and Secondary Schools and other Associations in which it was arranged that a joint meeting of these several associations should be held yearly on the Friday and Saturday following the week of Thanksgiving. In order to make this meeting more profitable for members of these associations, there will be a joint committee to arrange the programs for a general meeting on Friday. The individual associations will hold their regular meetings on Saturday, thus enabling them to carry out their usual programs.

After the business meeting the following papers were presented:

Mr. Eugene M. Dow, Mechanic Arts High School, Boston, "Recent Series of English Texts on Mathematics."

Miss Alice M. Lord, High School, Portland, Me., "Neither Algebra nor Plane Geometry but Mathematics."

Professor Emeritus W. E. Byerly, Harvard University, "Teaching of Geomery."

Professor H. N. Davis, Harvard University, "Slide Rules, Old and New," with an exhibit of about 50 rules.

THE New York section of the Association of Teachers of Mathematics in the Middle States and Maryland held a meeting Friday, February 16, 1917, in Ceremonial Hall of the Ethical Culture School with the following program:

1. "First Year Mathematics for High Schools," by C. Burton Walsh, of the Ethical Culture School.
2. "How Can We Minimize the Influence of Examinations upon the Teaching of Mathematics," by James H. Shipley, of the High School of Commerce.
3. General Discussion.
4. Exhibit of Instruments made and used by Pupils in the Ninth Year.

#### SECTIONS.

##### NEW YORK:

*Chairman*—ERNEST H. KOCH, JR., 155 W. 165th St., New York.

*Secretary*—EVELYN WALKER, 35 W. 82d St., New York.

##### PHILADELPHIA:

*Chairman*—JACOB B. KRAUSE, 3037 N. Broad St., Philadelphia, Pa.

*Secretary*—RUTH MUNHALL, 236 Harvey St., Germantown, Pa.

##### PITTSBURGH:

*Chairman*—CLYDE S. ATCHISON, 102 S. Wade Ave., Washington, Pa.

*Secretary*—SARAH L. BREENE, 6 Roselawn Terrace, Pittsburgh, Pa.

##### ROCHESTER:

*Chairman*—HENRY J. LATHROP, Brockport Normal, Brockport, N. Y.

*Secretary*—ARTHUR SULLIVAN GALE, University of Rochester, Rochester, N. Y.

##### SYRACUSE:

*Chairman*—ARTHUR E. NEWTON, Utica Free Academy, Utica, N. Y.

*Secretary*—FLOY A. ELLIOTT, 125 Furman St., Syracuse, N. Y.

EXAMINATIONS FOR TEACHING POSITIONS IN THE PHILADEL-  
PHIA HIGH SCHOOLS FOR BOYS AND HIGH SCHOOLS  
FOR GIRLS.

These positions (Day and Evening) are filled by appointment from Eligible Lists.

Examinations are held from time to time as occasion demands. The next for high schools for boys will occur about March 24, and for high schools for girls about March 10.

Applicants will receive due notice of the examination when the date has been determined.

The names of those who receive an average of seventy or over will be placed upon the Eligible List for a period of two years, at the end of which time the period of eligibility may be extended for one year upon the written application of the candidate.

*Preliminary Requirements.*

To secure the necessary card of admission to the examination, the applicant must complete the following arrangements at least seven days prior to the examination, in person or by mail:

Age limit 50 years.

(a) File a formal application on the official blank furnished by this office.

(b) File a certificate of physical fitness made out on the official blank furnished by this office.

(c) For MATHEMATICS, present proof of graduation from an approved college, or equivalent education.

A certificate from the president, dean, registrar or principal giving date of graduation, with degree, and bearing the college seal will be accepted in lieu of diplomas. Copies of credentials will *not* be accepted. Diplomas, etc., cannot be returned by registered mail unless the applicant furnishes the necessary postage.

*Scope of Examination.*

The examination will be partly written and partly oral:

1. *Oral Examination.*
2. *Methods of Teaching and Management.*
3. *The Technical Examination* for each candidate will be re-

stricted to the subjects of the department in which he desires to teach, and subjects necessarily related thereto.

4. *Mathematics*.—Algebra; plane and solid geometry; plane and spherical trigonometry; analytical geometry; calculus.

*Note*.—The names of candidates who enroll but do not attend the examination will not be continued on file unless the application be renewed.

Persons who fail twice are not eligible for further examination. The results of the examination will be mailed to each candidate.

Address all communications regarding the examination to the examiner, Keystone Building, Nineteenth Street above Chestnut, Philadelphia.

G. W. FLOUNDERS, *Examiner*.

#### NEW MEMBERS.

Clara J. Everett, 32 Barnes St., Gouverneur, N. Y.  
 Miss Edna Noyes, 51 Main St., Binghamton, N. Y.  
 Mr. R. C. Ireland, Massena, N. Y.  
 Mr. P. J. Bentley, Watkins, N. Y.  
 Nellie B. Retan, Eastwood, N. Y.  
 Terese R. Rosenthal, 223 W. 112th St., New York, N. Y.  
 Isabel G. Winslow, 1051 Tinton Ave., New York, N. Y.  
 Laura Landau, 316 W. 97th St., New York, N. Y.  
 Morris L. Bergman, 183 2d St., New York, N. Y.  
 Edith M. Morris, 396 E. 171st St., New York, N. Y.  
 Josephine Brand, 501 W. 138th St., New York, N. Y.  
 Albert C. Lutz, Vienna Agricultural H. S., Vienna, Md.  
 George C. Harter, Delaware College, Newark, Del.  
 Mr. O. H. Bruce, Westernport, Md.  
 C. Edward Bender, Oakland, Md.  
 John S. Hill, Stockton H. S., Stockton, Md.  
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# THE MATHEMATICS TEACHER

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## MATHEMATICS CONTESTS.\*

By ERNEST H. KOCH, JR.

From time immemorial games have been a necessary complement to the more austere forms of intellectual and social development. We have historic records of contests which enable us to obtain a familiar acquaintance with the manners and customs of nations as well as their achievements. To some of us these ancient games were only exhibitions of physical prowess while to others the staging of the orators and actors was a surpassing achievement. In our day we see these contest ideas culminating in great world expositions, art exhibitions, commercial shows and last but not least in the community masques. Cities vie with cities in resetting the contest motive so that each successful affair surpasses its predecessor in beauty and magnificence. International world fairs have permeated the nations, states, municipalities, colleges and schools with the spirit of the contest.

It is our purpose to show how the idea of a school contest has taken an intellectual form although modeled and staged after the physical counterparts on the track and field.

"What dire offence from amorous causes springs,  
What mighty contests rise from trivial things!"

In the class room we may set apart a definite period during which the pupils may participate in various games, such as

\* Presented at the Baltimore meeting of the Association, December 2, 1916.

checkers, chess or any of the many so-called children's games. These are intended more for the individual than for the group. All games are events of skill in the manipulation of fixed pieces, cards, devices or even individuals in which the element of distribution or motion is only partially under mental control owing to the limitations imposed by the rules of the game. A scholastic contest is a matching of intellects in a set task in which the written or verbal arts of expression play a minor rôle. All contests have their stimulus in the award of medals, banners and the publicity of honors. Any school subject may be used to afford material for a contest. In a commercial school the commercial subjects should have the greatest prominence. A Three "R" Contest is a competitive exhibition in the fundamentals of commercial education. This unique form of school activity was instituted for the purpose of interesting the public and the parents in the work of the school and for promoting the interest of the pupils in those studies which form the foundation of their life work. A program of the events of such a contest follows:

1. Addition
2. Subtraction
3. Multiplication
4. Division
5. Relay—four fundamental operations combined
6. Scissors—a crisscross in addition
7. Commercial geography
8. Spelling
9. Current events
10. Penmanship
11. Typewriting
12. Stenography

Such a program will give entertainment for two hours and offers the best reply and refutation to that part of the public which so glibly accuses the public schools of graduating poorer material than was graduated years ago. It should be observed that the mathematics takes up half of the program. These programs are altered by substitution of other subjects in the course, such as algebra, geometry, interest, bookkeeping, letter-writing, Spanish and public speaking. Contests take various forms, depending upon the method of public exhibition and demonstra-

tion and also upon the special requirements in the preliminary contests of elimination.

In the March, 1916, number of the *MATHEMATICS TEACHER* and June, 1916, number of *School Science and Mathematics* Mr. John H. McCormick and I have presented an account of a mathematics relay which has been tried in many secondary and elementary schools. The speaker who follows me will illustrate another form of relay which was originated and introduced independently in the William Penn High School of Philadelphia. The latter has been applied to both arithmetic and algebra. One year ago I learned of another independent contribution which was introduced in the Chicago schools. It is evident that these spontaneous and independent efforts in mathematical contests obey the same law of evolution which has given simultaneous and independent birth to identical inventions and to other forms of contemporary expressions of scientific, social and economic thought.

"Great contest follows, and much learned dust  
Involves the combatants; each claiming truth,  
And truth disclaiming both."

With these general introductory remarks I shall proceed to describe the mathematics relay as it has been developed at the High School of Commerce in New York City. A relay consists of four related parts presented in the following order: a continued addition, a continued subtraction, a continued multiplication and a long division. Four pupils constitute a team. Each member of the team performs and completes one of the four fundamental operations described above. This will be illustrated at the blackboard to make the situation and the performance clear. (Through the courtesy of the Eastern High School of Baltimore several teams of girls demonstrated the relay.) In the addition work two addends are written at a given signal. From these seven, eight or ten addends are formed according to the instructions. Each new addend is formed by adding the two preceding ones. All the addends are then summed and when the total is correct the subtractor proceeds and subtracts a given subtrahend five times in succession. He in turn is followed by the multiplier and the latter by the divider according to their respective assignments. The team finishing first is declared the

winner. The mechanics for handling the situation is described in the article referred to.

It is possible to get the entire class interested in the relays and in each of the four fundamental operations any one of which lends itself to the contest idea. In a very short time you will find groups of pupils proficient in one or more of the fundamentals. The entire class is divided into teams of four with a captain for each team. Every member of the class can do one of the operations well and is therefore on a team. The class competition becomes so keen that a representative team is soon chosen together with an alternate team to represent the class. These are often replaced through failure to win from a challenging team of the class. A round robin is arranged for the classes in the building. The most successful of these teams represents the building. The next step is the inter-annex contest and this leads to an all school team ready to meet all comers from other schools in the city. The final contest is an intercity contest but plans for this are yet in the making.

The relay always serves as an excellent drill in the class room and for outside assignment as a very simple change in the end number of the first addend makes all of the examples different yet very easy for quick verification on the part of the teacher. The subtrahend may be chosen so as to have a definite relation to the addends. Likewise the divisor may be chosen so as to have some definite relation to the multipliers. It has been found to be better to use nine addends or twelve addends after the ten addend work has been developed. If we designate the two addends by  $x$  and  $y$ , we have the following new addends and their subtotals. A number of ingenious combinations may be made from these algebraic relations so as to get an answer quickly from the original addends. Note the formation of coefficients.

	Addends	Subtotals
(1)	$x$	$x$
(2)	$y$	$x + y$
(3)	$x + y$	$2x + 2y$
(4)	$x + 2y$	$3x + 4y$
(5)	$2x + 3y$	$5x + 7y$
(6)	$3x + 5y$	$8x + 12y$

(7)	$5x + 8y$	$13x + 20y$
(8)	$8x + 13y$	$21x + 33y$
(9)	$13x + 21y$	$34x + 54y$
(10)	$21x + 34y$	$55x + 88y$
(11)	$34x + 55y$	$89x + 143y$
(12)	$55x + 89y$	$144x + 232y$
(13)	$89x + 144y$	$234x + 376y$
(14)	$144x + 233y$	$378x + 699y$

In the following examples for relay work twelve addends are required and the subtrahend is used five times in succession.

(1)	73659 85673 30483032 sum sub. 5196783 4499117 rem. mul. 9, 7, 6 1700666226 prod. div. 189 8998234 quot.	(2)	98967 83958 33729504 sum sub. 5431896 6570024 rem. mul. 9, 8, 7 3311292096 prod. div. 756 4380016 quot.
(3)	87657 98976 35585040 sum sub. 5638972 7390180 rem. mul. 9, 8, 6 3192557760 prod. div. 864 3695090 quot.	(4)	89678 56783 26087288 sum sub. 4658937 2792603 rem. mul. 9, 8, 7 1407471912 prod. div. 504 2792603 quot.

In these contests it will be found advantageous to use squared paper or square ruled blackboards.

Another highly interesting series of contests is known as the Scissors or Criss-cross. This may be applied to the fundamental operations involving integers, fractions or decimals in the same manner as the relay and can be used for horizontal as well as vertical addition and subtraction.

## THE SCISSORS APPLIED TO ADDITION.

A team of two pupils is designated "A" and "B." A and B begin work simultaneously at a given signal. They receive their respective assignments on library cards and transcribe these to the blackboard and then "blaze away." A is assigned addends (1) and (2) whereas B is assigned addends (10) and (11). They add in the relay fashion forming addends by adding each addend to the preceding one until eight addends have been written. The eight addends are summed. A and B interchange places, check their partner's work and then write their sum under that of the partner. They proceed in the new place making new addends until six have been written, then these are summed. A and B again interchange places, check and add their own last total to the partner's total, observing that the grand totals agree. The two pair of initial addends are composed of different numbers as shown below:

A begins here		B begins here	
(1 or $x$ )	396	these two addends are assigned	693 (10 or $w$ )
(2 " $y$ )	487	" " " " "	784 (11 or $z$ )
(3)	883		1477 (12)
(4)	1370		2261 (13)
(5)	2253		3738 (14)
(6)	3623		5999 (15)
(7)	5876		9737 (16)
(8)	<u>9499</u>		<u>15736</u> (17)
B continues here		A continues here	
(9)	24387	A & B interchange places, check	40425 (18)
(18)	40425	partner's work, transfer sums	24387 ( 9)
(19)	64812		64812 (24)
(20)	105237		89199 (25)
(21)	179049		154011 (26)
(22)	275286		243210 (27)
(23)	680196	A & B interchange places, check	<u>616044</u> (28)
A resumes here		B resumes here	
(28)	616044	partner's work, transfer sums	<u>680196</u> (23)
(30)	1296240	and add	<u>1296240</u> (29)

The answer always ends in a cipher and is 20 times the sum of (9) and (18). An increase of 1 in either (1) or (10) increases the answer by 420. An increase of 1 in either (2) or (11) increases the answer by 660. The final sum equals  $420(x+w) + 660(y+z)$ . Therefore if  $x$  is increased by the same amount which is subtracted from  $w$  the answer remains unchanged. This is likewise true for the combination  $y+z$ . It is this observation that enables us to set any number of addend pairs which will produce a given result, obviating copying of work by pupils.

If the addends are chosen so that  $(x+w)=1000$  and also  $(y+z)=1000$  the answer reduces to 1080000 which may be used as a key by means of which a number of examples may be set without effort as shown below:

$$\begin{array}{rclcl} x & 306 & 694 & w & x+w=1000 \\ y & 487 & 513 & z & y+z=1000 \end{array} \quad \text{sum} = 1080000$$

If the addends are chosen so as to form pairs of complementary numbers which are multiples of 100 another set of examples may be formed as follows:

$$\begin{array}{rclcl} x & 396 & 603 & w & \\ x+p & 404 & 707 & w+q & \\ \text{sum} & & = 1080(x+w) + 660(p+q) & & = 1190640 \end{array}$$

If  $(x+w)=1000$  then this reduces to  $1080000 + 660(p+q)$  as shown below:

$$\begin{array}{rclcl} x & 396 & 604 & w & x+w=1000 \\ x+p & 404 & 796 & w+q & p+q=8+192=200 \\ \text{sum} & & = 1080000 + 660(200) & & = 1212000 \end{array}$$

The following examples are appended for reference:  
Examples whose sum = 1080000:

$$\begin{array}{cccc} 396 & 604 & 396 & 604 & 397 & 693 & 395 & 605 \\ 487 & 513, & 488 & 512, & 489 & 511, & 480 & 520, \end{array}$$

Examples whose sum = 1080660:

$$\begin{array}{cccc} 396 & 604 & 396 & 604 & 397 & 693 & 395 & 605 \\ 487 & 514, & 488 & 513, & 489 & 512, & 480 & 521, \end{array}$$

Examples whose sum = 1298400:

397	694	396	695	398	693	397	694
488	785,	487	786,	489	784,	487	786,

Examples whose sum = 1299060:

415	676	415	676	417	674	416	675
489	785,	488	786,	488	786,	487	787,

Examples whose sum = 3459060:

1415	1676	1415	1676	1417	1674	1416	1675
1489	1785,	1488	1786,	1488	1786,	1487	1787,

Examples whose sum = 345906:

141.5	167.6	141.5	167.6	141.7	167.4
148.9	178.5,	148.8	178.6,	148.8	178.6,

It may be possible to prevail upon the editors of the Mathematics Teacher to give space for the publication of the activities of an intercity mathematics club. This space could be devoted to the activities of mathematics clubs, notices of contests and results. Under the auspices of the mathematics teacher and the local school organizations an intercity contest could be conducted by telephone or by having teams visit other city teams. The pleasure attending such a visit would prove a desirable incentive for a very active participation in the tryout contests. Arrangements could be made so that the expense of such a trip would not exceed the carfare for travel. The members of visiting teams would be distributed and entertained by the teachers of that school which acts as host.

"In their games children are actors, architects, and poets, and sometimes musical composers as well."

HIGH SCHOOL OF COMMERCE,  
NEW YORK CITY.

## RETURNS TO THE QUESTIONER.

By C. C. GROVE.

There still are those who think the professor has an easy time and proceed to help make him earn his "large" salary by asking him questions, with or without a stamp for reply. It seems to be expected that he is a cistern if not a spring, but they forget at what expense a good cistern is built and a gushing spring forced.

Two types of problems have come in rather frequently so that it seems worth while to make them known as a class so that when their simplicity is perceived they need not come any more for solution.

First a simple one: What must I pay annually, beginning at the date of purchase, to pay for a property valued at  $V$  dollars and interest at the end of each year at  $r$  per cent. for unpaid amount, so that all is paid at the expiration of  $t$  years?

*The method of solution is to set down the situation at the end of each year in symbolic form and from analogy write out the situation at the end of the transaction; thus,*

At end of first year  $(V - A)(1.or) - A$ . Amount of unpaid money less  $A$ .

At end of second year  $V(1.or)^2 - A(1.or^2 + 1.or + 1)$ . Amount of former balance less  $A$ .

At end of third year  $V(1.or)^3 - A(1.or^3 + 1.or^2 + 1.or + 1)$ .

At end of  $t$ th year  $V(1.or)^t - A(1.or^t + 1.or^{t-1} + \dots + 1.or + 1) = 0$ .

Multiplying and dividing the series after  $A$  by  $(1.or - 1)$ , we get

$$V(1.or)^t - A \frac{(1.or)^{t+1} - 1}{1.or - 1} = 0.$$

By taking logarithms of  $1.or$ , multiplying by  $t$  and  $(t + 1)$ , and finding the antilogarithms of the products, with ease the

value of  $A$  is found by substituting in

$$A = \frac{.0r \times (1.0r)^t}{(1.0r)^{t+1} - 1} V.$$

Second, a more complicated problem: A city wishes to issue 6 per cent. bonds. It can raise  $A$  dollars annually to pay interest on the bonds and establish a sinking fund that shall yield 4 per cent. and mature the bonds in 40 years.

1st. When  $A$  is available to start sinking fund at time of issuing the bonds.

Let  $B$  be the amount of bonds to be issued.

At end of first year,  $A(1.04) + A - .06B$ .

At end of second year,  $A(1.04)^2 + A(1.04) + A - [.06B] [(1.04) + 1]$ .

At end of third year,  $A[(1.04)^3 + (1.04)^2 + (1.04) + 1] - [.06B][(1.04)^2 + (1.04) + 1]$ .

At end of fortieth year, after multiplying and dividing by  $(1.04 - 1) = .04$ , we have,

$$A \frac{(1.04)^{41} - 1}{.04} - .06 \frac{(1.04)^{40} - 1}{.04} B - B = 0.$$

To show how simply this is done, we write out

$$\log 1.04 = .017033,$$

$$\log 1.04^{40} = .681320. \therefore 1.04^{40} = 4.8010,$$

$$\log 1.04^{41} = .698353. \therefore 1.04^{41} = 4.99306.$$

For those afraid of logarithms it is not bad to take the square times the square to get the fourth power; the fourth by the fourth then by the second power to get the tenth; the tenth by the tenth and finally the twentieth by the twentieth; especially if you multiply only for the figures that will become significant in the result, dropping off the end figures.

From the above figures and formula, we have,

	.04 3.8010
	95.025
	.06
	5.70150
	1
.04 3.99306	
99.8265A =	6.7015B

From this you determine either the bond issue for a given tax levy, or the tax levy  $A$  necessary to issue a given amount of bonds.

Second, in case the tax levy  $A$  is available only when the first interest on the bonds is due the formula may easily be seen to be

$$A \frac{(1.04)^{40} - 1}{.04} - \left[ .06 \frac{(1.04)^{40} - 1}{.04} + 1 \right] B = 0.$$

When  $A$  is \$20,000 the respective values of  $B$  are \$297.922 and \$283.593.

Although these formulas are to be found in texts on investments, the reader is usually not led to see how he can adjust them to his own immediate needs, or devise other formulas. This fact and the presentation of a concise form of solution furnish excuse for this note.

The second class of problems that come, deals with the theory of probability in its native state, *i. e.*, respecting games. The question sent in "to decide a wager" was: In a game of auction pinochle, the deck containing 48 cards, two of each color from 9 to ace, three players each holding 15 cards and 3 in the blind, what chance has a player, who has neither of the two aces of hearts in his hand, of drawing one ace of hearts from the blind?

After inquiry about games, of which the writer is profoundly ignorant, it is known that the solution may be made most clear to anyone knowing the elements of the theory, as follows:

Probability of getting a hand without either ace of hearts is

$$\frac{{}_{46}C_{15}}{{}_{48}C_{15}} = \frac{33 \cdot 32}{48 \cdot 47} = \frac{22}{47}.$$

Probability of getting 3 in the blind including at least one ace of hearts is

$$\frac{{}_{33}C_3 - {}_{31}C_3}{{}_{33}C_3} = \frac{31 \cdot 31 \cdot 30}{33 \cdot 32 \cdot 31} = \frac{31}{176}.$$

Probability that both these events happen is the product of

their probabilities, which is  $\frac{31}{376}$ .

To others it may suffice to put

$$\frac{{}_{46}P_{15}({}_{22}P_3 - {}_{31}P_3)}{{}_{48}P_{18}} = \frac{31}{376}.$$

COLUMBIA UNIVERSITY,  
NEW YORK CITY.

## THE ORDER OF TEACHING THE PARTS OF THE CALCULUS.

BY JOHN K. LAMOND.

When the average student reaches the calculus his ideas of a function, a variable, a variable approaching a limit, etc., are so vague, and oftentimes so entirely wrong, that it would seem wise, since the foundations of the calculus rest so largely on the theory of limits, to spend enough time at the beginning of the course to develop the theory of limits in a careful and thorough manner.

It is the present tendency to mix the differential and integral calculus. That is, to develop the two divisions side by side. Theoretically this may seem like an ideal thing to do, for the student will be made to see the interrelation of the two divisions of the subject from the very start. But in practice, since the ideas of the calculus are so new to the student, and so very much bigger than anything which he has encountered during his previous mathematical experience, it seems doubtful if the *average student* gains anything from such a treatment. Certainly he loses nothing, if the formulas for differentiating are developed first, with no mention made of the inverse operation, and the subject is not so likely to become confusing to the poorer students.

Having developed these formulas, the thing that the student is interested in is not another series of inverse operations. The thing he is continually asking himself and his instructor is, "What is this derivative, this thing we have spent several weeks in learning how to find, really good for now that we have it?" This natural curiosity is amply satisfied by maxima and minima, and rates. Having finished these subjects, if the other courses which the student is taking are such that some knowledge of integral calculus seems desirable, the indefinite integral, or both the indefinite and definite integral, may be very profitably introduced at this time, leaving the remaining topics of the differential calculus for later treatment.

From this point it seems doubtful if one can say just what is

best. Each individual teacher, knowing the capabilities and needs of his class, is the best judge of what to omit, and what to give, and the order of the giving.

PENNSYLVANIA COLLEGE,  
GETTYSBURG, PA.

BY C. C. GROVE.

The purpose of the committee in setting this topic is not quite clear. All that I can say, it seems, must have occurred to every one that has taught the subject. Some of the orders of presenting any school subject may be termed the historical order, the logical order, and the psychological order.

In the case of the calculus, the historical order is twofold according to the point of view or of the time and place of beginning. Despite this dilemma for us, there is much to be said in favor of an historical order of presenting a subject. It seems to fit into the student's growing capacities. He likes to see any living thing develop. In mathematics, however, it would sometimes be quite at variance with the logical order into which we so naturally go the more a subject becomes crystallized in our minds.

Last evening a friend was speaking of the piano and organ as means of musical expression. He felt the piano superior because its impulses of tone, its discontinuous notes, are suggestive and excite the imagination; whereas, the sustained notes of the organ melt into one grand finished product that we quietly admire. It seems to me that the logical order is rather like the organ, and that it is quite easy to have a presentation so logically coherent, and so naturally and simply and clearly knit together that the student feels nothing remains to be said or done, and yet becomes hopelessly lost when he tries to reproduce that "simple" presentation. The student's mind too needs to traverse some, at least, of that devious path by which the instructor reached his present logical formulation.

The third order combines the former two with the consciousness of that living interaction between instructor and class that enables the topics to be brought up at the pedagogically critical moment. This will vary somewhat with every combination of teacher and class. Thus it is, in part, that so many dif-

ferent textbooks appear. The teacher must direct this procedure carefully and good results are likely to follow.

As to the differential and the integral parts of the calculus, several years of teaching according to each of five different texts have led me to prefer to develop all the differential formulas before introducing integration, as the reverse process and then as a summation. We find no difficulty in maintaining interest through the more formal part, or any part for that matter. There is a growing interest as new power is gained to attack problems. The development of the subject is continually reviewing the mathematics studied before and bringing up new applications of that former material that furnish true delight to the student.

COLUMBIA UNIVERSITY,  
NEW YORK CITY.

BY ROSS W. MARRIOTT.

The subject of to-day's discussion is one which demands the close consideration of all teachers of the calculus, and to my mind resolves itself into the question as to whether we should hold to the time-honored custom of the presentation of the calculus or whether we should fit it to the needs of the present-day student. It seems to me, for example, that to take up the whole of the differential processes without considering the process inverse, is as unnatural as it would be in arithmetic or algebra to treat multiplication of all types of numbers before considering the operation of division, which I believe is seldom if ever done. Just as we sometimes treat division as a multiplication process, so we have occasion at times to treat integration as a process of differentiation. The processes of differentiation and its inverse are so closely allied that I believe there is an advantage gained by studying the elementary standard forms of the integration at the time we study the differential forms. Such modes of integration as require a transformation process could well be left to study under the integral calculus proper.

I think the order in which the calculus was invented has had a great deal to do with the manner in which it has been presented. When the differential calculus was invented it was found that the inverse process gave results identical with the older integral

calculus, which depended in no way upon the differential, hence the formal division into the two branches of calculus.

The demand for the calculus is continually growing, and comes from a varied class of students. Such subjects as engineering, chemistry, economics, and biology all have a claim upon calculus as an auxiliary, but many of the students of these subjects cannot afford to give time to an extended study of calculus.

If, then, we find that the calculus by a different mode of presentation better meets the needs of our students, we are justified in making such a presentation of its parts.

The arrangement of the parts of the calculus taught depends upon the object or end for which the student takes the calculus. It must be so selected that the work does not degenerate into mere mechanical routine, while at the same time the student becomes well grounded in the formal processes which are so necessary for the intelligent application of any branch of mathematics.

Simple practical application of the elementary portions should be introduced very early, the geometrical and physical significance of the derivative as soon as it is defined, and problems relating to it, may be introduced. Some of the elementary portions of curve tracing, maximum and minimum and rates can be taken up with advantage along with the formal processes of differentiation.

However, the practicality may be over done in the early stages of the calculus, and the student may lose sight of the significance of the formal processes, and so never be able to make much use of the calculus, as the real applications require a thorough notion of the formal processes.

A certain well-accepted textbook on calculus gives the symbol for an inverse circular function, and then states in italics that it is the angle whose sine is so and so. Shortly afterwards it makes an application in which this function enters additively with a pure scalar number. A student not long ago asked how it were possible to add them and get the measure of an area. This is what I call too much practicality or rather too much without sufficient preliminary formality.

I do not think such subjects as Taylor's theorem, theorem of mean, expansion of functions, etc., which are primarily applica-

tions of the differential calculus should be left until all the formal processes of integration are completed, as is done in so many textbooks, for these things have as wide an application as the calculus proper, and are indeed necessary for a logical and rigorous development of the calculus.

The significance of the calculus, then, the possibility of applying it in other fields, in short, its usefulness as an instrument should be kept constantly before the student during the study of the subject rather than deferred to some indefinite future.

A well-known educator has said that there is no more vicious educational practice, nor scarcely any more common one, than that of keeping the student in the dark as to the end and purpose of his work, for it breeds indifference and despair. The significance and usefulness of the calculus should not be kept from the student by following a time-honored custom, as the mysteries of a secret society are kept from the initiate until he has mastered the preceding degrees.

SWARTHMORE COLLEGE,  
SWARTHMORE, PA.

SHOULD ARITHMETIC BE TAUGHT TO ALL PUPILS  
IN THE HIGH SCHOOL? WHEN? HOW MUCH  
TIME SHOULD BE GIVEN TO IT?

BY FRANK H. SCOBAY.

There is in my opinion no doubt whatever but that some arithmetic should be taught to all pupils in the high school.

I do not know that there is any well-defined opposition to such opinion, but I do know of some who are not in favor of a review which covers the ground of arithmetic in the same way as it was done in the grammar school and with such objection to arithmetic in the high school I am entirely in sympathy.

In nearly all of our New Jersey school systems arithmetic is taught in the eight grades of the elementary school and judging from the students who come to our normal school about 25 per cent. of these have reviewed the arithmetic of the seventh and eighth grades sometime during the four years of the high-school course.

It would seem as though eight years is long enough to spend upon this beginning branch of mathematics without carrying the subject into the high school. It would be were it not that parts of the subject are beyond the mental grasp or maturity of mind of the pupil at the time they are presented. I do not think that little children of the first and second grades, as a rule, can comprehend the abstractions of number or pure number relations. These children would make more rapid progress if the study of the facts and processes of number were begun two years later. The next four years should be devoted to perfecting them in accuracy and a reasonable degree of rapidity in the fundamental processes, fractions, decimals and the elements of percentage with just enough rationalization of these processes and application to their surroundings or environment to lead them to understand and appreciate the purpose of arithmetic.

These are the years when children are most interested in mechanical processes. It is, when all things are considered, the

period during which they make the most rapid progress in calculation with reasonably large numbers.

In the seventh and eighth grades, that is those which immediately precede the high school, there should be an enrichment of the course in the way of applications. These I briefly mention under the heads of mensuration of some of the planes and solids, taking those which can be made concrete through the use of simple apparatus; applications of percentage to such business as we suppose a pupil of these grades can understand, such as profit or loss, a little of commission; taxes in connection with town expenses and such other very simple applications of arithmetic to social and industrial life as will appeal to their experience, emphasizing any application of community interest. Topics that pertain to investments of money, stocks and bonds, bank discount and exchange are often meaningless particularly when these exercises have no better bases than those afforded by the definitions and meager information of the textbook. But even where these topics are well taught I often hear my pupils say: "I never understood stocks and bonds or bank discount from my study of them in the grammar school."

The place for these topics of arithmetic is in the high school when the pupils can bring to them more maturity of mind and when they may often be correlated with topics of like nature and which belong in the high school. For illustration: If pupils take up a commercial course in the high school the study of discount should be in connection with commercial paper; stocks and bonds with the study of business associations or corporations. Just as the mathematics of the school shop, of domestic science, of agriculture are best studied where these activities are carried on. Even if these subjects are not included in the high school it is better to wait until such a time as the student has sufficient maturity of mind to understand something about the conditions upon which they are based and this is not before the high-school age. Probably the later they can be deferred in the high school the better.

Such topics as Euclid's method of highest common divisor and least common multiple of decimal numbers should be eliminated from arithmetic. Many teachers prefer to retain these on account of their supposed discipline. Whatever we may think

about this they should be taught in connection with the literal processes in algebra. If nothing more practical than a good test of the power to multiply and divide correctly they do afford that.

Cube and square roots of decimal numbers are more easily rationalized if associated with like algebraic processes. I do not think these topics have a place below the high school.

The progressions were always a part of the older arithmetic but have long been relegated to their place in algebra.

Many applications in percentage may be made a part of algebra, where the use of  $X$  for the unknown quantity facilitates or abbreviates the process. Some of you can go back with me to Olney's complete algebra, which devoted a considerable portion of the book to the topics of percentage and its applications. Some of these are obsolete now, but the plan of making algebra an instrument for generalizing the processes of arithmetic is a good one.

When a teacher in the high school some years ago, it was the custom to set aside a period of the last semester for a review of arithmetic. While this is the plan of which I do not approve I refer to it merely for the purpose of remarking that the students who were preparing for college brought to it a maturity of mind and a consequent interest because of this better understanding.

Many of our high-school students enter our normal schools, where the work should be that of adapting the subject matter of arithmetic to the grades of the elementary school and studying as far as possible the method of teaching it. It is a great handicap and one of general complaint in normal schools that our pupils do not understand the subject matter of arithmetic. Time must be taken to teach the arithmetic that should have been acquired in high school.

While I believe that all pupils in the high school should be taught some arithmetic it is better if possible that its applications be made to new fields. Correlations should be made as indicated with the higher branches of mathematics, with commercial and other pre-vocational subjects, with the physical and economic sciences.

As the use of arithmetic is to make concrete or determine the

quantitative side of these subjects the time for teaching it must always be wherever the opportunity arises and in close connection with the subject itself. The amount of time devoted to it can only be determined by the nature of the subject with which it is associated and the need of the pupil but the drill in the use of figures should be just as thorough and the pupil be made as efficient as though a term were set apart for the study of arithmetic.

STATE NORMAL SCHOOL,  
TRENTON, N. J.

BY AMY L. CLAPP.

Evidently, we agree as to the child's great and lamentable ignorance of arithmetic—the only question to be considered concerns the remedy to be applied. The most obvious one is "to give all pupils arithmetic during their first term in the high school." This seems hardly efficient, for, besides discouraging the pupil by repeating exactly the same subject that she has had, and often disliked, in the elementary school, we should also have to use the same methods that have been used before. Can we expect that our training, unlike that of the faithful elementary teacher, will endure permanently?

It seems to me that it would be more strategic to approach the subject from a different angle, that of algebra, and, besides gaining the increased interest due to a new subject to shape the course that it will definitely help the situation in arithmetic.

The pupil's weakness in arithmetic can be classed under two heads:

1. Lack of general mathematical common sense.
2. Inability to calculate quickly and accurately.

This first includes many things—among them is ability to read the problem and to reason. I need not try to prove to this group that algebra will help here in short word problems, that will teach the pupil to think more clearly. Then the pupil's ignorance of "short cuts" and slowness to comprehend and use them when taught can be helped if she is shown how they depend upon algebraic principles—*e. g.*,  $678 \times 245 - 678 \times 145$ ; or  $51 \times 49$ . Ease in solution of percentage problems can be increased if the equation is used. Lastly, the pupil's knowledge or rather lack of knowledge of fractions can be helped if we follow the ex-

ample of those teachers who use algebra explicitly to cast light on arithmetic. In teaching algebraic fractions, they refer to the, theoretically, familiar arithmetical fractions, and lose no opportunity to review fractions by giving both numerical substitutions involving fractions and also equations to be checked that have fractional roots.

This last touches on what I think is the most serious phase of the whole situation—the inability to calculate quickly and accurately, and, what is far worse, the habit of the pupil to pride herself on the fact that she cannot, for instance, add. She seems to regard the elementary operations as childish and quite beneath the notice of a person of her advanced age. In our school, we are trying to correct this by giving to all our commercial girls a daily drill in accuracy. At present, half of these are using the Courtis Practise Tests, and the results are so good with this half that hereafter we expect to do all our drilling by means of these. This regular drill accomplishes two things, first, it increases their accuracy and speed, second, and more important, it is subtly changing their attitude towards such work. My own little beginners in algebra were really mortified the other day when they made careless mistakes in adding  $+27$  and  $-19$ —a welcome change from the high-school student's usual attitude! Next term, we expect to give to all our pupils entering from the grammar school, regardless of their course, this same drill in the Courtis Practise Tests.

So much for the question of arithmetic during the first year of the high school—the decision as to whether a girl is later to take it depends, I think, largely on what she intends to do after graduation. Some definitely need it for their future training—*e. g.*, the commercial girls must take commercial arithmetic in their junior year in connection with their bookkeeping, and those girls preparing to go to the normal school must have a half year of arithmetic during their senior. But generally, it seems as if, with this first year drill, a girl could, more profitably, spend her time elsewhere in mathematics than in arithmetic.

SOUTH PHILADELPHIA HIGH SCHOOL FOR GIRLS,  
PHILADELPHIA, PA.

BY RUTH MUNHALL

To the first of these queries I would answer yes—decidedly yes. Arithmetic should be taught in the high school, and if the courses were not so crowded I should say to all pupils; for I have found almost without exception that the chief difficulty that besets the girl in algebra is an inability to perform simple arithmetical operations correctly; and that most of the failures are due to inaccuracy—and a good stiff course in arithmetic would go far to remedy this one besetting sin common to nearly all pupils.

But of course I fully realize that the time given to one subject is limited and that my desire to give all girls a good stiff course must be modified so I will take the subject up in four divisions; dealing separately with the four courses that we offer in the Philadelphia high schools.

*First. The College Preparatory Course.*—Here, we all realize, the work is very heavy, but it seems to me that we could slip in a little practice in old-fashioned mental arithmetic, which would be of great benefit to them all. It would help them to think more quickly and more accurately. A five-minute drill each time the class meets would be possible; in fact I am trying it in my senior class and the girls seem to enjoy it; as yet it is too early for me to say how profitable it will prove to them, but I am pretty sure that it will be worth the effort. Whether the senior year is the best place or not for this work I cannot say, but under present conditions it seems the logical place to put it.

*Second. The General or Normal School Preparatory Course.*—With us these girls do have a half year of arithmetic, but it seems to me that a longer course would be advisable for these girls are to be the future teachers of the children whom we will eventually get and if we could impress upon these potential teachers the importance of arithmetic we would in a roundabout way be preparing better material for the high school, as far as mathematics is concerned. This work ought to be in the senior year and ought to continue for a whole year. But just here we meet with the fact that an algebra review is given to the general girls for the first half of the senior year. And this is necessary, but would it not be possible to give these girls five hours of mathematics instead of three and thus give a longer time to the

arithmetic, either alternate the subjects or put the algebra into the first third and allow the arithmetic to take the last two thirds of the year.

*Thirdly. The Domestic Science or Home Economics Course.*—These girls should have a course in arithmetic during the first or second year—preferably the second. This course should comprise simple computations such as bills, budgets, household accounts of all sorts, measurements, estimation of the amount of material needed—given the dimensions, drill work for accuracy, some formula work. A very good thing would be to have a course in the senior year open to these girls. In this course more difficult work could be taken up and they, having more mature minds, would be able to better realize how much they need the work.

And last but by no means the least important comes the commercial course and there is one thing about which I have strong convictions, and that is that it should not be given in the first year. I have taught it to both freshmen and sophomore classes and I can truly say that the freshman class is not able to take in what the second year pupils can. I want a year of preparatory mathematics—algebra—then a year of good solid commercial arithmetic with plenty of drill work to try and develop accuracy. Then if the course can permit of it, it would be well to give these commercial girls a chance to have a term of arithmetic in their senior year. This course to treat of some of the harder commercial transactions. If this course is required put off teaching building association, stocks and bonds, and kindred subjects until the senior year. I have never tried this but I think it would work out well for these subjects seem hard for the majority of the girls in the first years, since they have no knowledge of business and lack this foundation, which they acquire as they take up their commercial subjects.

In looking back it seems to me that I have asked for a good deal but I am sure not for more than is needed. And I can reiterate my first statement: Yes—decidedly yes. Arithmetic ought to be taught to all pupils in the high school.

HIGH SCHOOL,  
GERMANTOWN, PA.

## MATHEMATICS CLUBS.

BY LOUISA M. WEBSTER.

Believing that courses in mathematics are second to none in value as a mental discipline, it seems meet that the teacher's best efforts may be profitably spent in devising plans which will stimulate a desire for research. I speak from personal experience when I say that one of our most difficult problems lies in making provision for the many points of a crowded curriculum which must be treated lightly, assigned for outside work or entirely omitted.

I remember, a most forceful lesson on roulettes was given as a club paper by a student whose time was not limited. She had done considerable reading, her facts were arranged systematically, she reduced much of her reasoning to the level of the lower-class students, and treated the subject more exhaustively than could have been done during the time allotted to one recitation.

That much valuable information is crowded out of the curriculum, that many most important facts are pushed aside from classroom work, that the time allowed to lectures is far too short to satisfactorily cover the majority of points which even the average student would find interesting are well-known and much-to-be-regretted conditions. The Mathematics Club offers a remedy, and also an opportunity for considering attractive views of the subject which find no place in the classroom.

The Hunter College Mathematics Club was organized by Professor Requa "as the result of a desire on the part of both the teaching and student bodies to investigate matters connected with mathematics, to study the phases of mathematical development which are crowded out of classroom work, and to keep the students in touch with the best thoughts of the times. It aims to be a source of profitable pleasure." That it has proven so all members will testify.

The meetings are held once a month from October to June. The first of each semester is largely a social function—a wel-

come to the freshmen. About every third year we invite, as guests of honor, the graduates of the mathematics department who have won some distinction. Our alumnae members are loyal. We seldom have a meeting that is not attended by several. Many are located in out-of-town schools. They write of their experiences, and this is helpful to the undergraduates; it shows them the practical side of the young teacher's work, the many rounds of the professional ladder, and the ever-increasing demand on the teacher's equipment. We take this as an evidence of their continued interest in the work, and their willingness to lend a helping hand.

Professor Requa says a few words at each meeting. Her talks offer suggestions for study or research or a comment on a magazine article or newspaper clipping. They always stimulate a desire for the further development of her topic. Every member of the teaching staff takes a deep interest in the work of the club. One serves as treasurer, collecting the 50 cents dues and acting as advisor for expenditures; one has general supervision, and others contribute papers and help wherever and whenever it is possible. While the topics are assigned to the students, each speaker works out her own subject matter. Usually the talks are given without notes, other than the citations of references. Illustration by models and blackboard drawings is extensively used. Inspiration and profit have also been derived from the talks given by men and women prominent in the mathematical field.

The president, vice-president, and secretary are members of the student body, chosen from the sophomore and junior grades, to serve a year. The candidates must have a record of superior scholarship and their interest in the club work must have been proven in some definite way, not merely by attendance at meetings. The president is always a junior who must have shown marked ability in her chosen field, also administrative powers. She hands the office to her successor as she reaches the upper senior grade. We consider the active co-operation of the students in the management of the club a valuable adjunct to their training for practical life. They become thoroughly impressed with the importance of study and research. Within the past three years eighteen of the graduates of the department have

taken their master's degree. Several who have done some graduate studying together presented the department with a set of books they had found specially useful, as an expression of appreciation of the benefit they have derived from the club.

Our club has been in existence eight years. It is a pleasure to report that two presidents have received high-school appointments, one is studying medicine, two have temporary assignments for college work, one is teaching in an elementary school, and another, now studying law, won the scholarship at New York University, when a member of the Woman's Law Class.

A list will show that the topics are chosen with reference to their mathematical or scientific interest.

The Parallel Axiom: Dr. F. Parthenia Lewis, Goucher College.

The Quadrature of the Circle: Dr. Elizabeth B. Cowley, Vassar College.

The Three Normals of the Parabola: Mr. John H. Denbigh, Morris High School, New York.

The Fourth Dimension: Dr. Feldman, Curtis High School, Staten Island.

The Application of Higher Mathematics to Business Principles: Dr. Schlaucht, High School of Commerce, New York.

Points on Which to Judge a Recitation in Mathematics: Dr. Breckenridge, Teachers' College.

The Benefits to be Derived from the Study of Mathematics: Dr. David L. Arnold, Julia Richman High School.

A Special Course in Geometry: Miss Matilda Auerbach, Ethical Culture School, New York City.

The Golden Age of Mathematics: Professor Emma M. Requa, Hunter College.

The Nature of Mathematical Knowledge: Professor Emma M. Requa, Hunter College.

Zero: Professor Julia Chellborg, Hunter College.

Calculating Machines: Professor Lao G. Simons, Hunter College.

Reports on Papers Read at Math. Assn. Meetings: Miss Evelyn Walker, Hunter College.

Old Mathematical Instruments: Miss Marcia Latham, Hunter High School.

Sonya Kovalsky: Miss Jean Robertson, Hunter High School.

The Geometry of Movement: Miss Martha Shott, Hunter High School.

Motion Pictures in Mathematics: Miss Grace Peters, Hunter High School.

Infinity: Miss Julia Simpson, a Hunter Alumna now on the Germantown High School staff.

Roulettes: Miss Edith Bainton, a Hunter Alumna of the Julia Richman staff.

The following have been given by student members:

The Mysticism of Numbers.

The History of Japanese Mathematics.

Baron Napier—His Life and Works.

What Women have Accomplished in Science.

Digital Reckoning.

Mathematical Fallacies.

The History of Time-pieces.

The Transit.

The Anaglyphs.

#### APPENDED LIST.

The Trisection of the Angle.

The Materials Used in the Teaching of Mathematics.

The Fundamental Principles of Universals.

The Russian Method of Multiplication.

The Different Proofs of the Pythagorean Formula.

The Properties of Number.

Algebraic Fallacies.

Non Euclidean Geometry.

The History of the Metric System.

Alligation.

Lincoln's Debt to Mathematics.

Magic Squares.

Ross's Blocks.

The Use of Imagination in Mathematics.

The Watch as a Compass.

The Life and Work of Galileo.

Mathematicians Who Have Become Famous in Other Fields.  
 Mathematics in Astronomy.  
 The Meaning of Billion.  
 Mathematics in Nature.  
 Number Games.

*"Every Little Movement."*

No longer does the college maid  
 Waste time or midnight oil,  
 Projections, trig. and logarithms  
 She's mastered without toil.  
 The higher math., all clear appears,  
 She sends all care away,  
 But integration is the thing  
 That holds the floor to-day—Ah!

Ev'ry little symbol has a meaning all its own,  
 Ev'ry integration by new formulæ are shown,  
 And hopeful feelings  
 That came a-stealing  
 O'er your being  
 Now fly despairing  
 As you work on, with some new methods,  
 Little methods, all, all your own.

It makes no difference, high or low,  
 Your "dates" go on the same;  
 When someone mentions tests are near  
 You work with might and main.  
 You take your seat in confidence,  
 But signs which once seemed clear  
 Now seem to mean a thousand things  
 Which cannot fit in here—Ah!

Ev'ry little symbol has a meaning all its own,  
 etc., etc.

L. HERTZ, 1910.

*O Dear! What Can the Matter Be?*

O dear! what can the matter be?  
 Dear, dear! what can the matter be?  
 O dear! what can the matter be?  
 This problem won't work out right!  
 We've struggled, we've juggled, equations we've buggled;  
 We've added, divided, subtracted—we tried it  
 Most sweetly, discreetly, and then we decided  
 To let it severely alone.

## Second Verse.

O dear! what can the matter be?  
Dear, dear! what can the matter be?  
O dear! what can the matter be?  
Th' ghost of the problem remains.  
It haunts us, it taunts us, with errors it flaunts us;  
We try to efface it, but mem'ry won't chase it—  
When, lo, inspiration!  
With courage we face it, and quickly we make it our own.

*Number Song.*

Tune—"Lulu Is Our Darling Pride."

Numbers are our pride and joy,  
Numbers great, numbers small;  
Juggle with them like a toy  
At our beck and call;  
Nor does vast infinity  
Puzzle or distress us;  
Nor can zero (hard to see!)  
In the least oppress us!  
Numbers are our pride and joy,  
Numbers great, numbers small;  
Juggle with them like a toy  
At our beck and call.

I know of no better way for a body of undergraduates to keep abreast of the times than through the medium of a well-organized club. It brings them in touch with the ever-increasing enrichment of their subject. The discussions furnish opportunities for the expression of individual ideas and for the application of theories. The importance of research and graduate work is emphasized. The members of the club show a deep interest in this phase of their education and we feel assured that most of them leave us with a feeling that they have been introduced to an extensive and interesting field of thought and labor.

Notices are given of the meetings of the several mathematical associations and the students are invited to attend. When the meetings are held in New York an official representative of Hunter College Mathematics Club attends and gives a report. Out-of-town meetings are reported by members of the teaching staff.

HUNTER COLLEGE,  
NEW YORK CITY.

## THE CONTENT OF A MATHEMATICAL COURSE FOR THE JUNIOR HIGH SCHOOL.\*

BY F. W. GENTLEMAN.

In view of the fact that junior high schools are actually being established in different parts of New England, it becomes the duty of this association to consider what shall be the nature of the work in mathematics for the course.

The junior-high-school period, in general, comprises the seventh, eighth and ninth school years, so the outline I am to present will be for a three-year course. One of the changes to be made for junior high schools is the gradual introduction of departmental work to bridge the gap from one-teacher to many-teacher instruction. This presupposes well-prepared teachers. Another change is the unification of the work in each subject, resulting in the establishment of a more connected, more logical, system. Hence it will be possible to offer much earlier some of the less difficult and more useful of the present high-school material, and to defer some of the more difficult and less useful material now offered in the elementary schools. If we attempt to saw off a strip of the present high-school course and nail it to a strip sawed off from the present elementary-school course and to claim thereby to have made a junior-high-school course, we are surely deceiving ourselves and defeating one of the main purposes, I believe, for which the junior high schools are being established. The change offers an opportunity for very necessary reforms in the content of the course and in the method of presentation.

Instruction in mathematics in the junior high school must necessarily begin where the pupil has reached as a result of the work in arithmetic for the first six years. It is commonly conceded that during this school period (Grades I.-VI.) he should have mastered the mechanics of arithmetic, and that the six

\* Read at the Springfield meeting of the Association of Teachers of Mathematics in New England, March 3, 1917.

years of school time is ample for the accomplishment of this purpose. By the mechanics of arithmetic I mean the processes of addition, subtraction, multiplication and division of integers, common fractions and decimal fractions. His attention thus far has been necessarily focused on the individual figures of numbers rather than on the number values so expressed.

In the mathematical work of the junior high school, the pupil should accustom himself to the standards of the business world; namely, that an example done once without review or check is only half done and that the responsibility for the correctness of the work must rest with the computer. The first of these means that the pupil must be shown, and compelled to use systematically, some method of reviewing or checking up his work. The second means that the responsibility for the correctness of his work must fall upon the pupil himself, and not upon some authority over him.

In the operation of addition and subtraction of numbers, no result should be accepted which does not carry with it the evidence of having been checked up. An accurate check in multiplication is performed by interchanging the multiplier and the multiplicand. An accurate check in division is performed by multiplying the quotient by the divisor and adding the remainder.

In the operation of multiplication and division, results should be estimated before any computations are performed. These estimates should be part and parcel of all the work submitted. This means that emphasis should be placed upon rational or common sense methods of locating the decimal point. Furthermore, if the method of multiplication taught were one in which the figures of the multiplier were used from left to right, then the approximation and the mechanics would go hand in hand; since the decimal point is located in the first partial product and the succeeding partial products are added as corrections thereto. This method, in later scientific work where approximate numbers are involved, lends itself to a considerable economy in the number of figures used.

In all this work, and in that which follows, process and speed should not be stressed to the sacrifice of judgment regarding results, nor to the sacrifice of the accuracy of results. Again, the choice of problems for application here and later should be

more carefully considered than appears to have been done in many current textbooks. Problems coming from impossible conditions, and naming impossible prices, are to be avoided, as well as problems that are mere collections of words to make some process necessary. Advertised sales, household budgets, statistical reports, etc., offer legitimate sources for problems. Furthermore, if we are to ground the pupil thoroughly in the fundamentals of arithmetic, so that the business world will be better satisfied with our product, we should make the wording of the problem simple and direct until the one principle involved in that problem has been mastered by the pupil. One of the main weaknesses in our problem work of to-day in the elementary course, is that many problems are so involved that the pupil becomes accustomed to failure and loses confidence in his ability to handle any arithmetical computations that require thought on his part.

Concerning the subject of percentage, from my teaching experience I am convinced that the presentation of its elements in the usual order of Cases I. and II. should be reversed. That is to say, that first the pupil should learn to interpret and to express, as per cent. relations, those relations between the number data in the fields familiar to his common experience. It is necessary for him to get a clear conception of the idea of per cent. as so many hundredths, or such a fractional part of a given amount. After the pupil has comprehended the per hundred idea and has realized that the per cent. obtained is a per cent. of some definite amount, then he may use intelligently the per centum idea in finding per cents of amounts; he is no longer in the dark as to the interpretation of the computation called for in these problems, so in making his computations he can proceed to apply common sense.

In considering the applications of percentage, it is important to note that the per centum idea has a much wider field of application than the monetary field alone. These applications should include problems concerning school data, comparisons of lines and of areas, and comparisons from the field of statistics, etc.

The third case, so-called, of percentage, the formal methods of computing interest, successive discounts, marking goods to

sell at a gain per cent. on the marked price, the computing of per cents in statistical work to certain degrees of accuracy, introductory treatment of taxes and insurance, should be deferred until the second year.

The equation should claim an early place in the course of the junior high school. It naturally occurs in the following topics: statements of elementary number-facts; formulas of mensuration, about which I shall speak at length later; statement of the equality of two ratios, here serving as a simple means of approach to the ratio idea involved in the per cent. relation; formulas for percentage, for interest and for scientific facts; and statement of general problems not included in any of the above, many of which problems have heretofore required a solution by the analytic method. The equation affords a simple direct method of expressing mathematical relations. By its use mathematical solutions are clarified. It is needed by the future mechanic and other tradesmen if they are to read trade journals intelligently, since the equation is the world's way of expressing a rule.

The kind of equation most needed by the pupil in the first and second year of the junior high school, is that which requires the axioms of multiplication and division for its solution. The treatment of the equation in these years should be natural and informal. The pupil should indicate definitely his progress, step by step, and should definitely assure himself of the correctness of his result by a check.

Another topic that should occupy an important place in the early part of the junior-high-school course is the measurement of the familiar geometrical figures and the drawing of these figures to full size and to scale. Familiarity with their shapes and properties, and a knowledge of the terms applied to them will remove many of the difficulties and misconceptions that are met later in the systematic study of geometry. For this work in measurement, the pupil should be supplied with a protractor for angle measurement, and with a ruler on which the inch is graduated to tenths as well as to sixteenths and the foot graduated to hundredths. With such a ruler measurements may be obtained in decimal fractions as well as common fractions, thereby extending the work in decimal computation beyond the monetary

field. Such a ruler would be especially useful in making scale drawings.

Special emphasis should be placed upon the care with which the measurements are made. Here may be made a study of the shape and properties of the square, rectangle, triangle, parallelogram and trapezoid. For the second year, this work should be extended to include the use of the compass for simple geometrical constructions, such as the drawing of perpendiculars, parallels, bisectors, etc. At this time the different plane figures should be systematically grouped and their common properties studied.

The graphical representation of number-data from the field of statistics has a place in the junior-high-school course. By comparing the lengths of lines and rectangles and parts of circles representing certain groups of facts, the pupil may learn to read number-data so expressed and also have experience in making graphs from given data. This work will assist him in reading intelligently the many articles appearing in magazines devoted to scientific and social problems. Furthermore, through such work he can visualize the idea of "round" number, one kind of approximate number.

The mensuration of the areas of the square, the rectangle, the parallelogram, the triangle, the trapezoid, and the circle is work within the comprehension of the first-year pupil. The rules for these should be developed from diagrams and these rules expressed as formulas at once. All this work should be on a rational basis, and a formula such as  $A = bh$ , should mean, first of all,  $A$  (the number of square units)  $= bh$  square units. For this the unit of area (a square something) must be visualized. We should be sure that the pupil carries away no such incorrect ideas as  $5 \text{ in.} \times 8 \text{ in.} = 40 \text{ sq. in.}$  For this work in mensuration the pupil should obtain as much of his data as possible by actual measurements, then his common sense and judgment may be better trained in the use of data that he may have in any problem. If this sort of training in computation be given him in the first year of the junior high school, then in the mensuration of the second year the distinction between a number obtained by count and one obtained by measurement may be made. On this foundation, approximate computations may be taken up and

the results retained to that number of figures that corresponds to the given data of the problem.

Accuracy (so-called) where results from measured data are required to several decimal places, gives the pupil the wrong idea of what is meant by mathematical accuracy. The pupil should be trained to realize what sort of an answer he ought to get, then make an effort to get the result correct to a degree that his common sense demands. When asked your age you do not say that you are 25 years, 3 months, 5 days, 4 hours, 8 minutes and 30 seconds old. Even were such an answer correct, it would be practically senseless. When such a kind of "accuracy" is required in answers, problems cease to be real and results to be of value.

This discussion concerning the rational kind of answer leads to the topic of square root, which should come in the second year. The one really rational method for getting square root is that known by some as Newton's method. The "completing the square" method quickly degenerates into the following of a mechanical rule. For Newton's method the knowledge of many squares becomes essential, the rational estimate is all-important, and the idea that the square root is one of the two equal factors of a number can not be lost sight of.

This is the natural place in the course to apply one of the most useful facts of geometry, the Pythagorean theorem. Following this, a study should be made of the development and use of the formulas for finding the surfaces and volumes of the block, the cube, the prism, the cylinder, the pyramid, and the cone. Here, too, some of the formulas based upon the simpler facts of elementary science might be introduced to advantage, as elementary science is likely to have a place in the course of the junior high school.

Near the end of the second year, equations involving the axioms for addition and subtraction should be studied, and this followed by a study of the solution of simultaneous linear equations by the method of elimination by addition or subtraction.

Throughout the course, the teacher should require that the work be neatly done; that clear, concise statements be made showing the progress of the work, when necessary; and that the computations be systematically arranged.

At the end of the first two years of the junior high school, the pupil, who has followed the course as outlined above, should be well grounded in the fundamentals of arithmetic, so that he can attack with confidence the problems that he is likely to meet, should he be forced to leave at the end of his eighth year. He should have a fairly clear idea of the shapes and properties of the common plane figures and solids. He should have some grasp of generalized arithmetic, fitting him to continue his work in his ninth year much more intelligently than he does at present.

In the last year of his junior-high-school course, he should again make a study of arithmetic, to get some idea of its unity and its general values; he should consider more extensively the applications of percentage to the field of science, getting some definite idea of the per cent. of error in data and in result, together with a more definite knowledge of the meaning of significant figures. He should make a somewhat more intensive study than before of such applications of percentage as taxes, bonds, mortgages, insurance, etc.

Once more he should deal with formulas, but this time with the transformation of those formulas already familiar to him, and with the building of new formulas from rules, whether these rules are the results of his own experience, or not; in other words, he should now be able to symbolize his scientific language.

The addition, subtraction, multiplication and division of polynomials should be studied as an aid in solving certain types of equations. A study should now be made of linear equations, involving the treatment of negative numbers; of simultaneous linear equations, including the graphical solution; of complete quadratic equations solved by factoring; and of simultaneous equations, one quadratic and one linear, solved by substitution and by graphs. Enough factoring would be necessary to make it possible to solve any quadratic having rational roots.

Near the end of this year, I would give the pupil an introduction into the systematic study of geometry, proving informally with him certain fundamental statements, and having him prove certain other statements formally, choosing for such formal demonstrations those statements the proofs for which are clear-cut and definite.

In conclusion, it seems to me that, if the course for the junior high school is arranged somewhat as I have outlined, and certain changes of method are made as outlined, then such results as the following might be expected:

A better understanding of quantitative relations;

A more common sense viewpoint concerning the value of results, with a growing respect for sensible accuracy;

More strongly developed habits of self-reliance;

A more thorough grounding in the fundamentals of arithmetical computation;

Earlier development of the power of independent reasoning in mathematical work;

For the pupil who leaves at the end of the eighth or ninth year, a greater ability to handle a variety of the mathematical tools used in the solution of the problems of everyday life;

For those who go to the senior high school, a course better fitted to suit their capacities and tendencies, since the teacher in charge would have wider opportunity than now to see the pupil's interest in, aptitude for, and capacity to grasp the mathematical point of view;

Finally, a stimulation of his interest in the further study of mathematics by giving him a clearer idea of what the subject is about, and by presenting to him a vision of the extent of the applications of mathematics to the different fields of the world's work;

For the pupil who enters the senior high school, less danger of unwise choice of mathematical work, because he already has a fair knowledge of his mathematical limitations;

Finally, a keener interest in the further study of mathematics, because he has a clearer idea of its meaning and a vision of its manifold applications to the world's important work.

#### PROPOSED OUTLINE.

##### *First Year.*

1. Review of fundamental operations of arithmetic.
2. Equations: (a)  $bx = c$ ; (b)  $\frac{x}{a} = \frac{b}{c}$  (ratio).
3. Measurement of straight lines, angles and plane rectilinear figures. Drawing to scale. Straight line graphs.

4. Percentage: (a) Finding what part one number is of another.  
(b) Finding percents of given amounts.  
(c) Applications: single discount, simple interest, commission.
5. Mensuration: (a) Areas of rectangular figures and the circle.  
(b) Rectangular and circular graphs.
6. Summary: Applied problems.

*Second Year.*

1. Review of fundamental operations of arithmetic applied to business transactions; approximate products.
2. Formulas: (a) Perimeters and areas of plane figures.  
(b) Square root; Pythagorean theorem.  
(c) Surfaces and volumes of solids.
3. Percentage: (a) Finding the base, percentage and rate given.  
(b) Applications: successive discounts, interest (formal method), notes, savings banks, taxes, insurance.
4. Construction of geometrical figures; classification of plane figures; graphical representation of statistics by broken line.
5. Equations: (a)  $ax + b = cx + d$ .  
(b) Simultaneous linear.

*Third Year.*

1. Review of arithmetic, to include bonds, mortgages, taxes, insurance, significant figures.
2. Formulas: (a) Transformation of formulas.  
(b) Construction of formulas.
3. Linear equations: (a) Involving addition and subtraction of polynomials.  
(b) Involving multiplication and division of polynomials.  
(c) Involving negative numbers.  
(d) Simultaneous (graphs).
4. Factoring: (a)  $aQ + bQ - cQ$ ; (b)  $Q^2 - O^2$ ; (c)  $Q^2 + 2QO + O^2$ ; (d)  $Q^2 + aQ + b$ ; (e)  $aQ^2 + bQ + c$ .

5. Quadratic equations: (*a*) Pure; (*b*) complete (solved by factoring); (*c*) simultaneous, one quadratic and one linear (solved by substitution and by graphs).
6. Introduction to systematic study of theorems of geometry.

MECHANIC ARTS HIGH SCHOOL,  
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## NEW BOOKS.

**Rosemary.** By ALICE E. ALLEN. Boston: The Page Co. Pp. 96. 50 cents.

A good story about twins and their schooling.

**Virginia of Elk Creek Valley.** By MARY ELLEN CHASE. Boston: The Page Co. Pp. 297. \$1.35 net.

This sequel to "The Girl From the Big Horn Country" is full of life, and contains many interesting characters.

**A Place in the Sun.** By MRS. HENRY BACKUS. Boston: The Page Co. Pp. 410. \$1.35 net.

This is an interesting story of an animated young girl who succeeds in spite of the disadvantages of birth, by placing great faith in the theory that here in America every one has a chance to win.

**The Barbarian.** By BREWER CORCORAN. Boston: The Page Co. Pp. 305. \$1.50.

The hero of this story was an ungainly country boy who goes to a boys' school and on account of a sensitive nature makes few friends at first. He has a good mind and is gradually led out into the interests of the school and other boys. A good, wholesome story.

**Analytic Geometry and Calculus.** By FREDERICK S. WOODS and FREDERICK H. BAILEY. Boston: Ginn and Co. Pp. 527. \$3.00.

The present work is a revision and abridgment of the authors' "Course in Mathematics for Students of Engineering and Applied Science." The condensation of a two-volume work into one volume has been made possible partly by the omission of some topics which can well be postponed to a later course, but largely through a rearrangement of subject matter and new methods of treatment.

The rearrangement of material is especially seen in the bringing together into the first part of the book of all methods for the graphical representation of functions of one variable both algebraic and transcendental. This has the effect of devoting the first part of the book to analytic geometry of two dimensions, that of three dimensions being treated later when it is required for the study of functions of two variables. The transition to the calculus is made early through the discussion of slope and area. From this point on the methods of analytic geometry and the calculus are intermingled.

Among the subjects omitted are determinants, much of the general theory of equations, polars, and diameters related to conics, evolutes,

complex numbers, and some types of differential equations. The book is intended primarily for first and second years in college or technical school. The number of problems offered, some two thousand in all, permits a variation of assignments from year to year.

**A Course in Mathematical Analysis.** By EDOUARD GOURSAT. Authorized translation by E. R. HEDRICK and OTTO DUNKEL. Boston: Ginn and Company. Volume II. Part II. Differential Equations. Pp. 300. \$2.75.

Since the appearance in translation of Volume I, this treatise has exercised an increasing influence on mathematics instruction in the United States and is now recognized as one of the standard reference texts. Its wide use is due not only to the reputation of its author but to its clarity of style, its wealth of material, and the thoroughness and rigor with which the subjects are presented.

The second volume, in its French form, has long been as well and as favorably known as the first. It has now been radically revised, and the present edition is a translation of the revised edition of the text. Both volumes, in the American edition, are distinguished by unusual excellence of typographic workmanship and careful accuracy of translation.

Volume I treats the subjects studied in a second course in calculus in American colleges and prepares the way for the study of the higher branches of analysis—notably differential equations and the theory of functions. Volume II, issued in two separately bound parts, each of which corresponds to a course in many American colleges, covers the theory of functions of a complex variable and the theory of differential equations.

**Laws of Physical Science.** By EDWIN F. NORTHUP. Philadelphia: J. B. Lippincott Company. Pp. 210. \$2.00 net.

The general propositions or laws of science, are the fundamental basis of all exact knowledge and mastery of physical forces, and their application. What man knows of the world he lives in, is dependent upon the recognition and application of these principles. Hitherto these have not been collected in one volume for ready reference and to enable the student and reader to obtain under a single view the very epitome of the world's heritage of exact knowledge. Professor Northup has performed a valuable service in filling this obvious gap in the literature of physical science.

The student in one branch of science, who has found it difficult to gain a knowledge of the important principles and facts in other branches, will find in this book exactly what he seeks to obtain—a broad view of the entire field of natural law. To those who wish to gain an intelligent grasp of our rich mental inheritance, without having the time or means to give years of study to the search, the "Laws of Physical Science" will come as a boon and a stimulus to further investigation and wider reading. The book is divided into six sections: mechanics; hydrostatics,

hydrodynamics and capillarity; sound; heat and physical chemistry; electricity and magnetism; light. Of pocket size and weight, the volume is admirably fitted to be the student's daily companion as an indispensable book of reference.

**Preliminary Mathematics.** By F. E. AUSTIN. Hanover, N. H.: the author. Pp. iv + 169. \$1.20.

While this book was originally written as a help for those who wished to improve their mathematics without going to school, it has been adapted for use as an auxiliary text. The first part is designed for junior high schools, the second section for high school use.

The author connects arithmetic and algebra, giving some excellent practice in the operations of arithmetic, and gradually bringing in the use of the algebraic notation and methods.

Throughout the book, and especially in the second section, the emphasis is placed on the solution of problems. This work is rather uneven, the analysis being well done in some cases, and poorly done in others.

**A Brief Account of Radio-Activity.** By FRANCIS P. VENABLE. Boston: D. C. Heath & Co. Pp. vi + 54.

Dr. Venable has written this book to supplement the meager treatment usually given in text books on chemistry. It covers the history of the discovery of radio-activity, the properties of the radiations, the changes taking place in the bodies, the alpha particle, a discussion of the atom, and a chapter on the connection of all these matters with the theory of chemistry.

**Vocational Mathematics for Girls.** By WILLIAM H. DOOLEY. Boston: D. C. Heath & Co. Pp. vi + 369.

This is the most complete attempt that has been made to prepare girls for the use of the mathematics needed in their business relations. A foundation is first laid by a review of arithmetic, with some extensions, such as training in reading plane and drawing to scale. Over one hundred pages are then given to the problems in home making, and the rest is divided between such subjects as "Dressmaking and Millinery," "The Office and the Store," "Arithmetic for Nurses" and "Problems on the Farm." In addition there is an excellent appendix containing material of general value, or of less common application to women's work.

The book gives the impression of not only teaching arithmetic, but combining it in interesting form with much valuable knowledge on other subjects.

## NOTES AND NEWS.

THE twenty-eighth meeting of the Association was held Saturday, April 28, 1917, at the junior school, Trenton, N. J. The morning session was opened at 10:45 with an address of welcome by Anna P. Hughes, vice-principal of the junior school. The topic for the morning was "Mathematics in the Junior High School." Very interesting papers on this topic were presented by the following speakers: William Betz, East High School, Rochester, N. Y.; Charles Barton Walsh, Ethical Culture School, New York City, N. Y.; Harrison E. Webb, Central High School, Newark, N. J.; Howard F. Hart, Montclair High School, Montclair, N. J.; Louise Northwood, Junior School, Trenton, N. J.

Between the morning and afternoon sessions there was an inspection of the new junior school.

In the afternoon session a very interesting paper on "Household Arts Arithmetic," written by Katharine F. Ball and Miriam E. West, of the High School, Plainfield, N. J., was read by Miss Ball.

The topic "Composite Courses in High School—Their Content, Their Strength, Their Weakness" was then discussed by the following speakers: George Alvin Snook, Frankford High School, Philadelphia; Martha W. Crow and Helen S. Opp, West Philadelphia High School for Girls; Margaret Groff, South Philadelphia High School for Girls.

The last topic of the afternoon, "Composite College Courses in Mathematics," was discussed by Floyd F. Decker, Syracuse University, Syracuse, N. Y.; William J. Fite, Columbia University, New York City.

At the close of the program Professor C. B. Upton, of Teachers College, New York City, gave a short and interesting talk on the subject of "Fusion in Mathematics in the Secondary Schools."

There were about 100 in attendance at the meeting and all present seemed to feel that it had been a very profitable meeting.